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Advancing Mathematical and Statistical Sciences in the Era of AI, Big Data, and Uncertainty

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Professor Dr. Zahoor Ahmad
Secretary General, ISOSS
and
Chairman, Department of Statistics
University of Sargodha

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FOREWORD

The Department of Statistics and the Department of Mathematics, University of Sargodha, in collaboration with the Islamic Society of Statistical Sciences, successfully organized the **22nd International Conference on Statistical Sciences (ICSS-2025)** on December 15-16, 2025, under the umbrella of Research Arena 2025. The conference was held under the theme ***"Advancing Mathematical and Statistical Sciences in the Era of Artificial Intelligence, Big Data, and Uncertainty"***.

The conference brought together leading academicians, researchers, practitioners, and students from around the world to exchange ideas and present innovative research in statistics, mathematics, and data science. It served as an important international platform for discussing theoretical advancements, methodological developments, and real-world applications in an increasingly data-driven and AI-oriented environment.

The scientific program covered a wide range of contemporary topics, including Big Data analytics, computational statistics, mathematical foundations of artificial intelligence and machine learning, Bayesian inference, stochastic processes, uncertainty quantification, environmental and health data sciences, spatio-temporal modeling, and applications in agriculture, economics, and social sciences. The conference featured approximately 95 research papers, along with 16 international keynote lectures and 12 national invited talks, reflecting its strong academic depth and diversity.

The Opening Ceremony was graced by distinguished dignitaries, including Prof. Dr. Iqrar Ahmad Khan, Chairman, Punjab Higher Education Commission (PHEC), as the Chief Guest. Senior leadership of the University of Sargodha, including the Vice Chancellor, Pro-Vice Chancellor, and Dean of Sciences, emphasized the critical role of mathematics and statistics in advancing artificial intelligence, data-driven research, and evidence-based decision-making.

A defining feature of ICSS-2025 was the strong integration of mathematics and statistics, demonstrating how mathematical theory provides the foundation for statistical modeling, while statistics transforms theoretical insights into practical tools for solving complex problems across science, industry, and society.

The conference witnessed active participation from scholars representing multiple countries, including the United Kingdom, China, Czech Republic, Australia, Bangladesh, Turkey, Saudi Arabia, the United Arab Emirates, Qatar, Oman, and Pakistan. The presence of internationally renowned keynote speakers and leading national experts greatly enriched the academic discussions and collaborative opportunities.

The conference also played a significant role in fostering international research collaboration and promoting emerging methodologies in AI-driven modeling, Bayesian inference, and data-intensive sciences. It provided a valuable platform for young researchers to engage with global experts and contribute to advancing quantitative research.

The organizers gratefully acknowledge the financial support of the Higher Education Commission (HEC) Pakistan, which facilitated international participation and contributed to the success of the conference. The organizers gratefully acknowledge the financial

support facilitated by Prof. Dr. Muhammad Hanif, President ISOSS, whose leadership and financial support were instrumental in the success of ICSS-2025. Appreciation is also extended to ISOSS, keynote and invited speakers, session chairs, participants, organizing committees, and volunteers for their dedicated efforts.

The papers included in these proceedings have been carefully peer-reviewed by experts in their respective fields, ensuring high academic quality and relevance. These contributions reflect current advancements and future directions in mathematical and statistical sciences.

The 22nd International Conference on Statistical Sciences (ICSS-2025) stands as a significant milestone in strengthening interdisciplinary research and reaffirming the central role of mathematics and statistics in shaping the future of artificial intelligence and data-driven innovation.

Professor Dr. Zahoor Ahmad
Secretary General, ISOSS
and
Chairman, Department of Statistics
University of Sargodha

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WAVELET INAUGURATION IN THE EMISSIONS OF NEUTRINOS FLUX FROM EXTRATERRESTRIAL REGION

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ABSTRACT

The existence of Neutrinos emission from extraterrestrial region indicated that this particle has been detected in various laboratories of the world. It has been observed that these laboratories are responsible for recording n laboratory is in Japan known as Kamikanado. We obtained set of observations from both the neutrinos emission originated from deep space. Among these labs one of the labs is situated near Sudbury, Ontario, Canada known as Sudbury Neutrino Observatory (SNO). Second well-known labs. It is obvious that the observations recorded at SNO have been utilized in this manuscript. It is essential to look at the inauguration aspects of neutrino behavior. This could be better understood by analyzing the primary illusion of the neutrino flux using HAAR wavelet with few levels.

For this purpose, we have utilized the SNO observed set- I of observations ranging from November 1999 to May 2001 of D_2O. We have included set-II of observations that varies from July 2001 to August 2003 of salt water with particulars recorded, Run start Time since midnight. The samples used in this paper are of 250 entries from both the observations sets. In this presentation the most spectacular exhibition of the neutrino flux in its frequency components of the flux reaching this biosphere. The inauguration of wavelet approach to present the hidden periodicities in the emission of neutrinos has been introduced

Thus, in this communication our aim and objective is to investigate the neutrinos affairs to introduce a new paradigm. We have opened a fresh field of analyzing, and modeling different aspects of neutrinos using wavelet approach. We intend to carry out research at the Institute of Space Science and Technology, University of Karachi with the collaboration of the Imam Ahmed Raza University of Emerging Sciences and Technologies. Also International neutrinos detection laboratories such as Sudbury Neutrino Observatory, Canada, and Second well known laboratory is in Japan collaborated this research in Pakistan. In this piece of information we are in a position to describe the neutrino emission characteristics investigated in real world. It has also been known from the literature survey that the same kind of work has not been framed in the third world countries where no neutrino detection laboratories exist.

This paper will provide beneficial aspects for the private and public organizations where the experts are trying to explore deep space emissions detection and their characterization like the current study. We intend to strengthen our work by assessing frequency components in the form of periodograms. We have analyzed the set of observations recorded at SNO.

KEYWORDS

Neutrino Flux; Deep Space, SNO detection station, Manifestation of Wavelet Contribution and Periodogram preview.

INTRODUCTION

It has been known for a long time that the basic structure of atom according to Rutherford comprises a nucleus having proton and neutron at the center of the atom and electron moving round the nucleus. It has been explored that during the 19th century the origin of neutrino is linked to the discovery of β decay of nuclei. At the same time after Solvay conference the famous Physicist Fermi established theory of beta decay and developed the structure of beta spectrum and the interpretation of rest mass of the neutrino [1-5].

Pauli during the year 1930 affirmed the familiarity of energy and momentum conservation of the process of β-decays via hypothesizing neutrino Neutrinos are known as neutral leptons that are not well measured since long and detection of neutrinos is difficult as neutrinos rarely interact with other matter. We have found that neutrino laboratories in the world are detecting neutrinos coming to earth from deep space. Solar neutrino research entered a new era in April 1996, when the Super-Kamiokande experiment began to operate [5-11]. In the previous era, solar neutrinos were detected by four outstanding experiments;

a) The radiochemical Homes take chlorine experiment, the Kamiokande water.
b) Cherenkov experiment, and the two radiochemical gallium experiments, GALLEX and SAGE.

In these four exploratory experiments, typically less than or of the order of 50 neutrino events were observed per year. In this communication, we intend to examine the spectral contributions of the neutrino emissions from deep space during specified periods and the hidden periodicities in the emission. We know that the Neutrinos were known as being leptons, are of three kinds called the electron neutrino, muon neutrino and the tau neutrino $(\nu_e, \nu_\mu, \nu_\tau)$.

The Neutrino detector station at Sudbury, Ontario, Canada called “Sudbury Neutrino Observatory (SNO)” where the detection of this particle is taken place and the collected samples of set of observations will be analyzed here in Pakistan by our research group.

Neutrino flux will be assessed using different scientific disciplines [12-18].

The Manifestations of Neutrino Emissions from different Processes

Nitrogen-Oxygen (CNO) cycle, nitrogen and oxygen are used as catalyst and then regenerated during reaction.

According to this cycle the following reactions are generated:

$$^{12}_{6}C + ^{1}_{1}H \rightarrow \ ^{13}_{7}N + \gamma$$

$$^{13}_{6}C + ^{1}_{1}H \rightarrow \ ^{14}_{7}N + \gamma$$

$$^{14}_{7}N + ^{1}_{1}H \rightarrow \ ^{15}_{8}O + \gamma$$

$$^{15}_{7}N + ^{1}_{1}H \rightarrow \ ^{12}_{6}C + ^{4}_{2}He$$

In the above reactions the neutrino flux has been indicated with the formation of helium nucleus. The presence of neutrinos from different perspectives is obvious. Therefore, we intend to study an inaugural preview of neutrinos emission from deep space during specified period [19-22].

Source of Observations

The Sudbury Neutrino Observatory recorded two sets of observations, first starting from 2nd Nov 1999 to 27th May 2001 using heavy water (D_2O) as interacting medium, having parameters recorded variables, Run start Time since midnight and Length of run, in Sec, having 559 entries and second set of observations starting from 26th July 2001 to 28th Aug 2003 using salt water as medium of interactions with same particulars recorded, having 1212 entries are taken. It is noted that for proper symmetric comparison and analysis, samples are so taken that coincide with solar year time starting from 2nd Nov 1999 for heavy water (D_2O) and 2nd Nov 2001 for salt water set of observations thereby celestial position of our Planet Earth that is orbiting round the Sun in the Galaxy should be same as possible at the time of recording of observations [23-25].

Physical Features of Neutrinos

It has been known that solar neutrinos have been detected at the Sudbury Neutrino Observatory (SNO) showing the decay of ^{8}B via the charged current (CC) reaction on deuterium and elastic scattering (ES) of electrons. This CC reaction seemed to be more sensitive in case of ν_e's and the ES reaction has sensitivity to ν_μ's and ν_τ's [23]. The report on the measurement of the total flux of active ^{8}B neutrino at SNO from the solar emission indicates neutrino flavor transformation. The CC and ES reaction rates are consistence with earlier results and with the NC reaction rate under the hypothesis of flavor transformation. It has been found that the ^{8}B neutrino flux is in agreement of Standard solar model. The abrupt changes on daily basis have been reported in the CC, NC, and ES at SNO with different reaction These results showed direct measurement of the day and night asymmetries in the neutrinos A Global fit to SNO's day and night energy spectra and set of observations from other solar neutrino experiments strongly favors the large mixing angle (LMA) neutrino oscillation analysis [24]. It has been investigated that at SNO the total flux of active ^{8}B neutrinos has been precisely measured from the Sun with energy dependence of the electron neutrino with their survival probability. According to the experimental result this flux has equivalency with calculated solar model. All three fluxes (CC, ES and NC) data from the pure heavy water D_2O phase presented and analyzed, were

in general derived under the assumption of an energy-independent survival probability of undistorted ^{8}B energy spectrum. The determination of results at SNO for salt fluxes also showed energy dependence and survival probability of same type of neutrino ν_e. SNO Collaboration based on correlations between salt results and its heavy water results, comments "there are correlations being the same detector, these are not large, since some changes have been made to both the detectors [24-26].

Implementation of Quantum Signal Processing Techniques

We intend to mention the complex nature of the emission that could be resolved using the unique technique for analyzing the signals. This technique is being utilized first time in this region of Pakistan for fractionating signals in their spectral components. In this communication we have recorded set of observations only for Sudbury neutrino observatory. In order to strengthen the above-mentioned facts, we are in a position to use the approach that can easily partition our neutrino signals coming from the solar bursts. These signals have been recorded as neutrino flux by the two well-known laboratories namely SNO, Canada and SKK, Japan.

We are now purposefully using signal processing technique known as "wavelet". This approach can be termed as an oscillation, like a wave that starts from zero amplitude increases and then falls back to zero. Wavelets can be combined with identified missing or damaged signal portions by convolution technique to extract information from the unknown portions [26-27].

It has been known that wavelets are finite-energy functions with localization properties. It does imply a short-term Fourier transform that can exhibits hidden periodicities in the complex signals such as neutrino emissions. It is also considered as mathematical microscope that can sketch small wave under the graph of the expression] is zero. In the spectral domain, this property is equivalent to, at $\omega = 0$. This spectral-domain behavior makes the wavelet a band pass filter (Chan & Liu 1998). The unified frame work supporting the wavelet transform that are developed independently. The concept of wavelet can extant the application in large areas, it treats both continuous and the discrete time cases. It provides very general techniques that can be applied to many tasks in signal processing and therefore has numerous potential applications [28-32]. We can now consider wavelet performance to cover the sudden variations or to functions which have simple discontinuities on smooth surfaces and which are smooth elsewhere [33-34].

It is to be noticed that the function ψ using translation and dilation is built that constitute the fundamental bits/ atoms expressed in Eq 4.1.

In this case the function of limited energy, its continuous wavelet transform is defined by the function given in Eq 4.2.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{\alpha}} \text{ where } \alpha \in R^{+}, b \in R \tag{4.1}$$

$$C_f(a,b) = \int_R f(t)\ \overline{\psi_{a,b}(t)dt} \tag{4.2}$$

Explanation on Wavelet function for local analysis

Here the coefficient Cf analyzing f with the wavelet ψ, function f is then defined by its wavelet coefficients $Cf(a,b)$, $a \in R+$ and $b \in R$.

They measure the fluctuations of function f at scale a.

The trend at scale a containing slower evolutions is essentially eliminated in $Cf(a,b)$.

The analysis in wavelets makes a local analysis of f possible, as well as the description of scale effects comparing the $Cf(a,b)$ for various values of a.

Indeed, let us suppose that ψ is zero outside of $[-M,+M]$, so is zero outside the interval $[-Ma+b, Ma+b]$.

Consequently, the value of $Cf(a,b)$ depends on the values off in a neighborhood of b with a length proportional to a.

THRESHOLDING

It has been known during few years that on the approach of wavelet characterization to process the signals of phenomena occurring in free space.

In this technique it is said that the act of thresholding is one of the most useful processing method.

The concept of eliminating or modifying small coefficients is termed as thresholding.

We know that noise is a typical aspect of signals therefore thresholding process is one of the most effective tool for reducing noise from signals and promoting signal recognition and in signal and image compression.

There are generally four types of thresholding techniques such as Hard, Soft, Quantile and Universal thresholding. In Table 1. these are exhibited with HAAR levels 1-8, Unscaled white noise, Scaled white noise and non-white noise.

The most popular are hard thresholding, and soft thresholding, whereas the coefficients are attenuated following a linear scheme.

Selection of thresholding methods depends on the application.

Therefore we have utilized the hard and soft thresholding for our research [35-38].

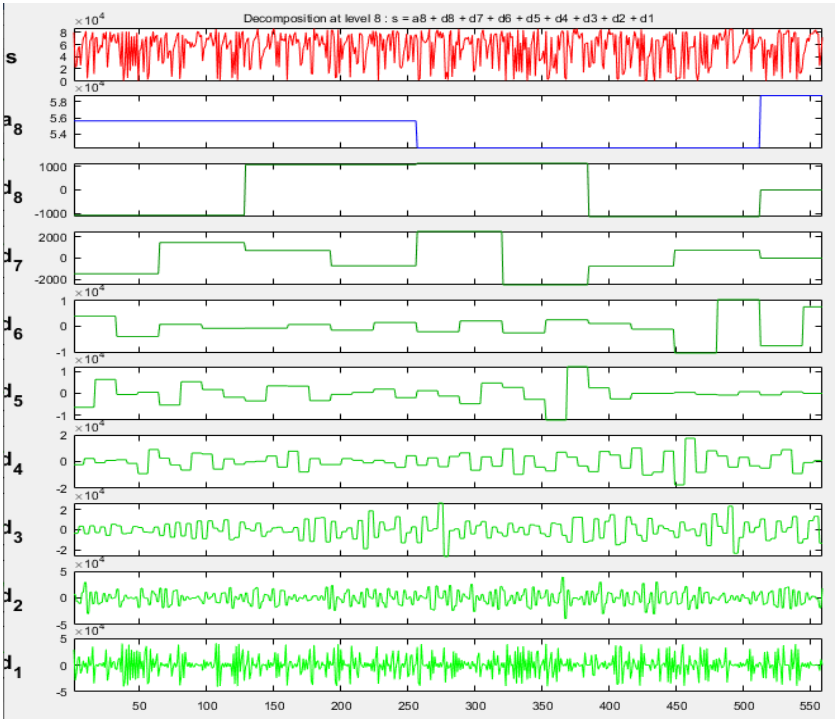
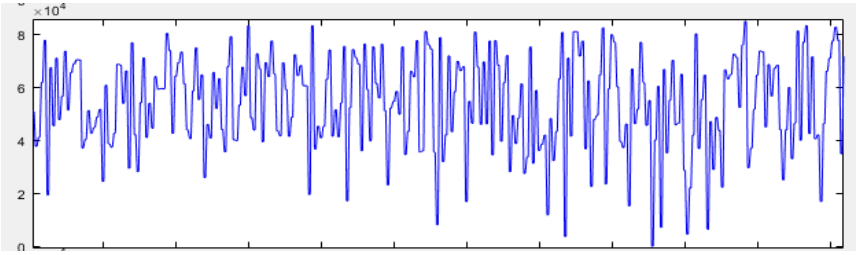


Figure 1, Combined Manifestation of Haar Wavelet Family, indicating Original Signal, Approximation and Decomposition at Level – 8 for SNO 1st Variable Run Start Time since Midnight in Sec of D_2O Set of Observations.

Depiction of approximation temporal behavior of SNO 1st Variable Run Start Time since mid-night midnight in Sec of D_2O set of observations.



Decomposition of temporal illustration of SNO 1st variable Run start Time since midnight in Sec of D_2O set of observations.

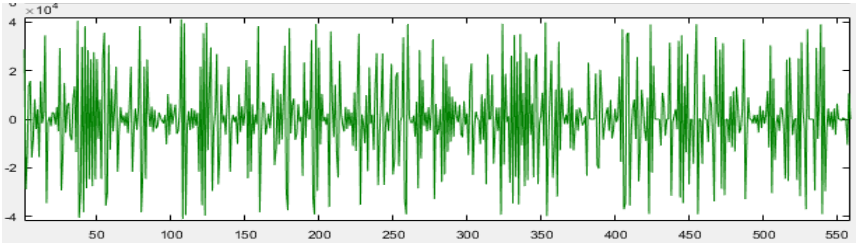
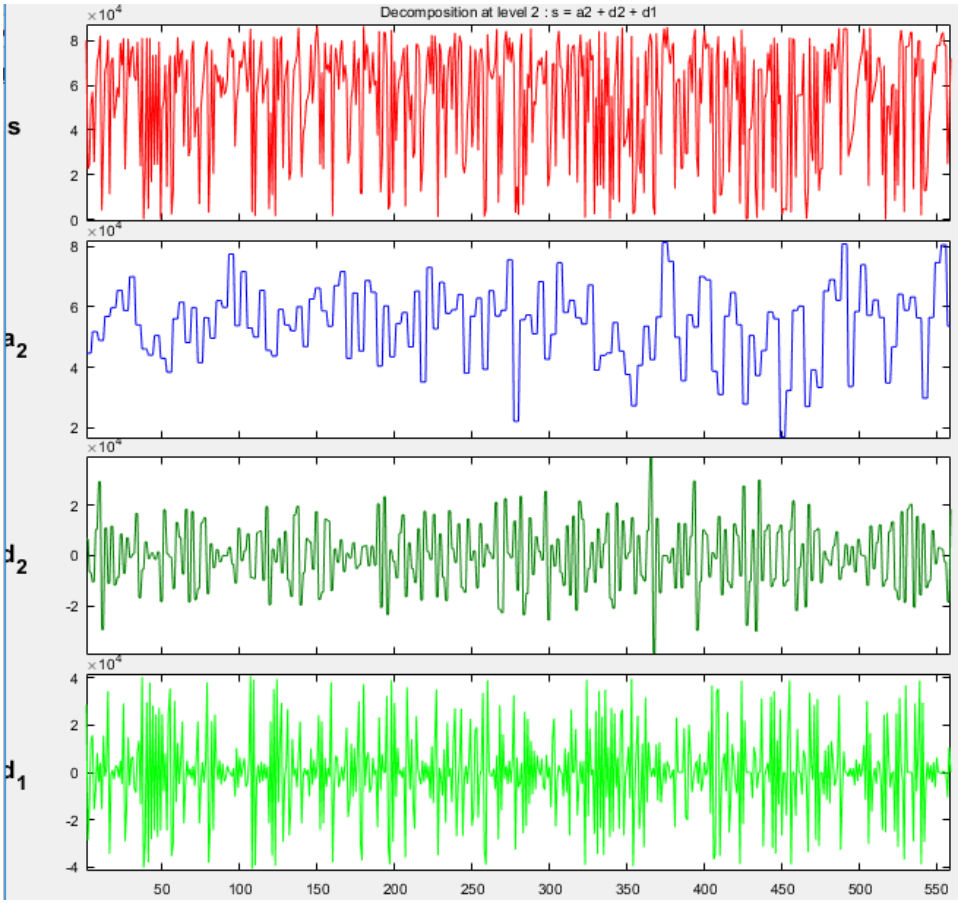
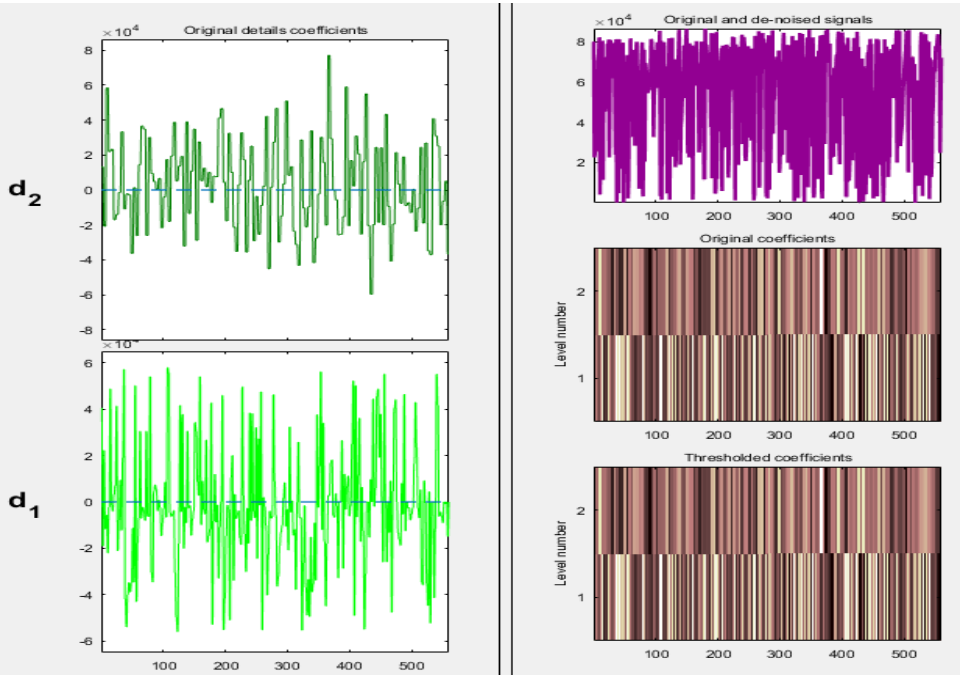


Figure 2, Illustration for Haar Wavelet Family, Set of Observations Signal, Decomposition (Approximation and Details) at Level – 2 for SNO 1st Variable Run Start Time Since





This Figure 2.a and b is Denoising representation of (Fixed Form threshold, Unscaled white noise) at level – 2 for Haar wavelet family for SNO 1st variable Run start Time since midnight in Sec of D_2O data indicating original and de-noised signal, Original coefficient, Threshold coefficient and Original detailed coefficients of level 1 & 2.

Table 1
Haar Wavelet Threshold Values of SNO D_2O Run Start Time since Midnight in Sec.

Haar Wavelet Soft & Hard Thresholding Method						
Threshold Value (Fixed Form)						
Level	Unscaled White Noise		Scaled White Noise		Nonwhite Noise	
	Soft	Hard	Soft	Hard	Soft	Hard
1.	3.558	3.558	58002.194	58002.194	58002.194	58002.194
2.	3.558	3.558	**63052.816**	**63052.816**	76917.255	76917.255
3.	3.558	3.558	**63052.816**	**63052.816**	75488.298	75488.298
4.	3.558	3.558	**63052.816**	**63052.816**	69778.428	69778.428
5.	3.558	3.558	**63052.816**	**63052.816**	56125.882	56125.882
6.	**3.558**	**3.558**	**63052.816**	**63052.816**	**82021.176**	**82021.176**
7.	3.558	3.558	28742.642	28742.642	28742.642	28742.642
8.	3.558	3.558	18135.998	18135.998	18135.998	18135.998

Figure 3, Denoising graphs (Fixed Form threshold, Scaled white noise) at level – 2 for Haar wavelet family for SNO 1st variable Run start Time since midnight in Sec of D_2O data indicating original and de-noised signal, Original coefficient, Threshold coefficient and Original detailed coefficients of level 1 & 2.

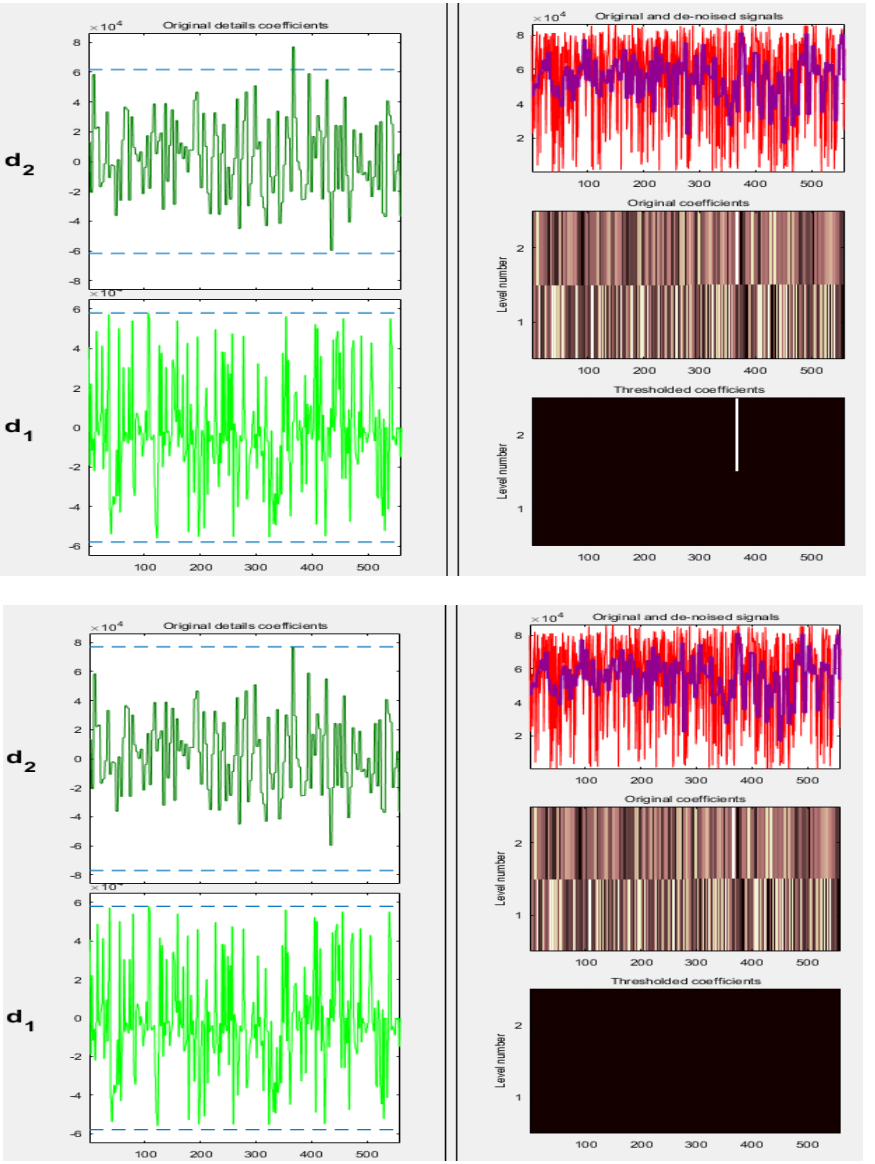


Figure 4-5, Statistical graph indicating original signal, histogram and cumulative histogram for Daubechies -9 (DB-9) wavelet family at level – 2 for SNO 1st variable Run start Time since midnight in Sec of D_2O data.

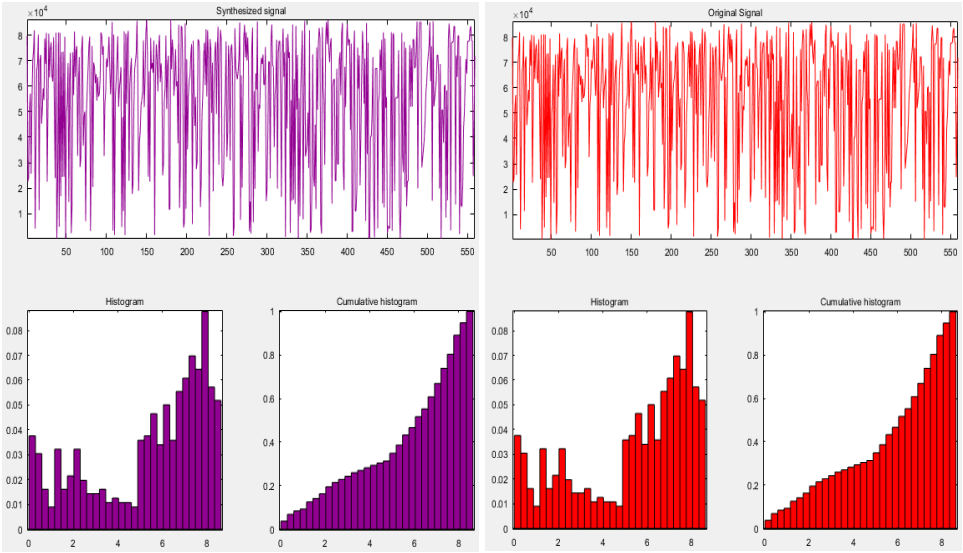


Figure 6, Statistical graph indicating Coefficient of Detail signal, histogram and cumulative histogram for Haar wavelet family at level – 1 for SNO 1st variable Run start Time since midnight in Sec of D_2O data.

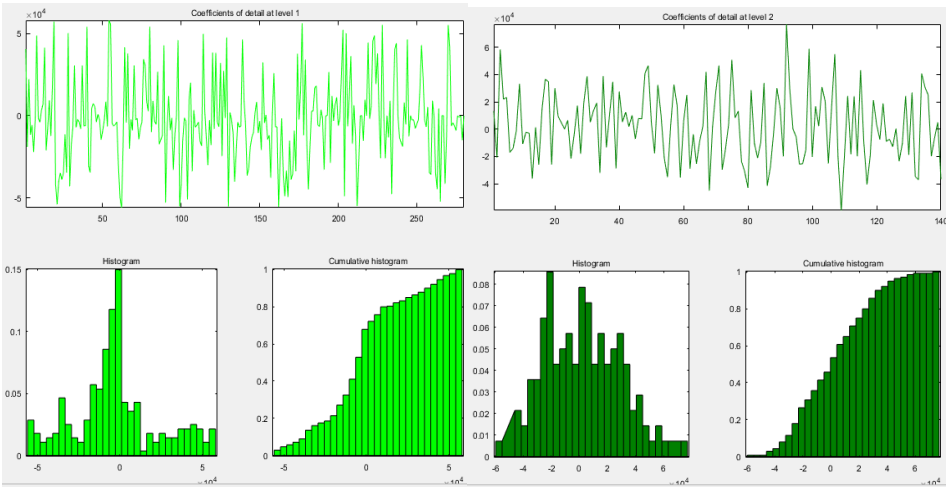
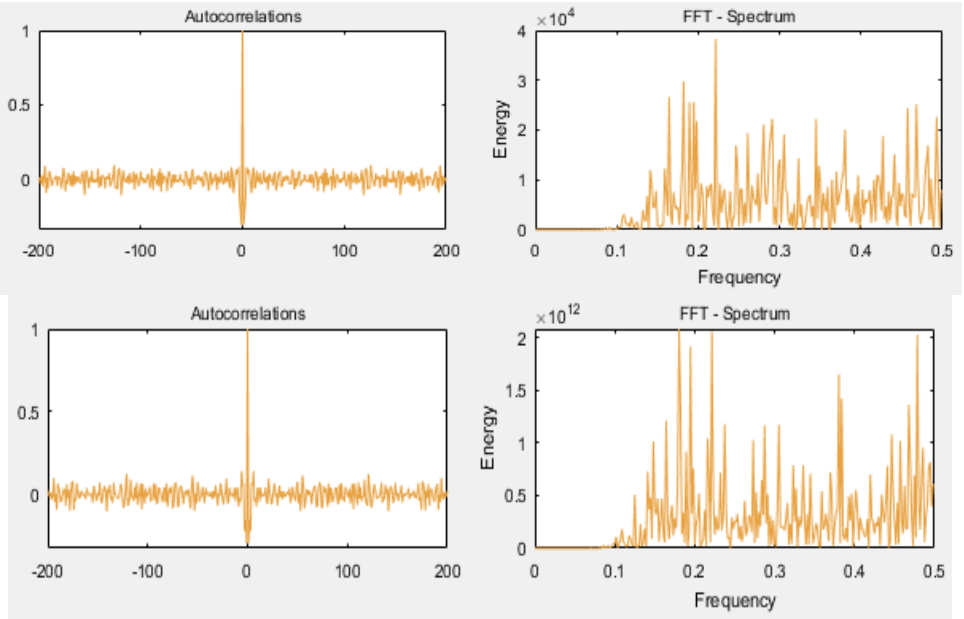


Figure 7, Autocorrelation Plot (left) and FFT spectrum (Shannon Energy Levels, right) of Denoised SNO 1st variable Run start Time since midnight in Sec by Soft Thresholding Method (Fixed Form Threshold) in Un-scaled White Noise of D_2O data using Daubechies -9 (DB-9) wavelet family at Level-2.



This Figure 8, Display Histogram and Cumulative Histogram of Residuals (Haar, Level-2) of Denoised SNO 1st variable Run start Time since midnight in Sec by Soft Thresholding Method (Mini max Threshold) in Scaled White Noise of D_2O data.

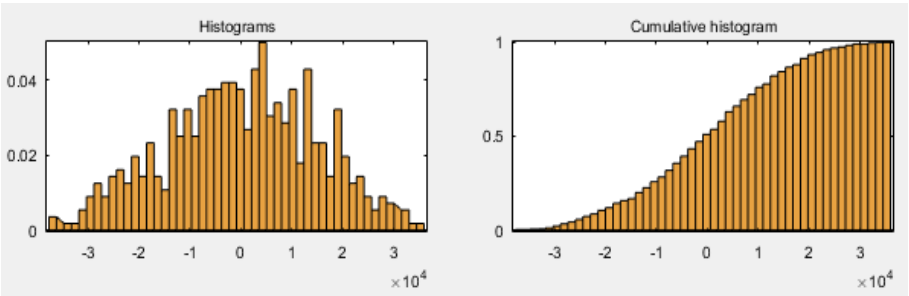


Illustration of Time Domain and Frequency domain for neutrino flux.

In this section parametric values of the neutrino emission can be computed indicating time domain and frequency-domain concepts.

These domains illustrate the spectral and periodogram analyses.

Time Series: $Xt = A\ Cos(2\pi\omega t + \phi)$

Useful trigonometric identity:

$$A\ Cos\ (2\pi\omega t + \phi) = \beta 1\ Cos\ (2\pi\omega t) + \beta 2 Sin(2\pi\omega t)$$

where $\beta 1 = A\ Cos(\phi)$ and $\beta 2 = -A\ Sin(\phi)$.

This identity is used when we determining the periodogram of a set of on

Mode of Computation of parametric values for neutrino flux

It has been examined that HAAR diagnostics are related to energies and frequencies contained in spectrum.

In the case of spectral analyses, the entire sets of observations have been decomposed into its sine wave components.

As regards the decomposition of a time-series into a sum of weighted sinusoidal functions is to utilize Fourier transform comprising discrete and continuous expressions.

Similarly a periodogram fascination is considered as an estimate of signal spectral density.

Sophistication of Periodogram is obvious if we plot amplitude vs. frequency characteristics of finite impulse response (FIR) filters and window functions.

It has been indicated that the Fast Fourier Transform (FFT) spectrum analyzers also utilized as a periodograms describing the time sequence.

From the knowledge of spectral analyses it has been examined that the power spectral density of a continuous function, $x(t)$ is the Fourier transform of its auto-correlation function.

$$F\{x(t) * x(-t)\} = X(f)X\hat{} * (f) = |X(f)|\hat{}2 \quad (1)$$

It has been pointed out that a time series can be expressed as a combination of cosine (or sine) waves with differing periods and amplitudes.

An examination of autocorrelation, partial autocorrelation function and line spectrum related to a time series also helps us in the detection of the underlying auto regression model.

It is said that if the autocorrelation function are exponentially decaying with one significant partial autocorrelation and line spectrum support (Low frequencies if the AR coefficient β_1 is positive, higher frequencies if β_1 is negative), then the under lying model is an AR(1) model.

It has been observed that 95% confidence interval is used to estimate autocorrelation (38-40)

Table 2
Stationary test for SNO Set of Observations I & II, Variable (Run Start Time since Midnight) of Heavy Water (D_2O) and Salt Water Parametric Values are Recorded

Augmented Dicky-Fuller Test Statistic		**D_2O Heavy Water**		**Salt Water**	
		T – Statistic	Probability	T – Statistic	Probability
		-16.97750	0.0000	-13.69809	0.0000
Test Critical Values	1% Level	-3.456514		-3.456514	
	5% Level	-2.872950		-2.872950	
	10% Level	-2.572925		-2.572925	

Table 3
Largest Periodogram Peaks of Time of Start of Run since Midnight of SNO D_2O Set of Observations

Variable	Run Start Time since Mid Night - Five Largest Value				
	(1)	(2)	(3)	(4)	(5)
Periodogram Peaks	4.56×10^9	3.72×10^9	3.64×10^9	3.58×10^9	3.47×10^9
Frequency	0.4880	0.1640	0.3840	0.2880	0.3360
Period	2.0492	6.0976	2.6042	3.4722	2.9762

Explanation on Table 1. of SNO D_2O Set of Observations

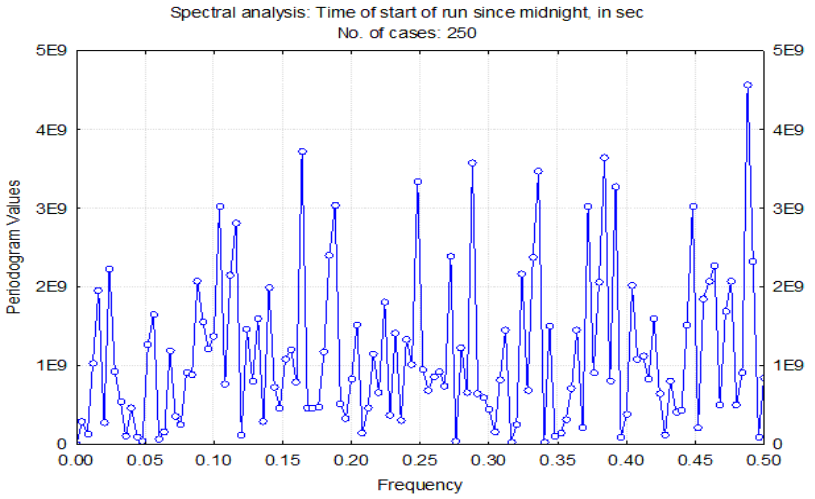
For Set of observations I, the ADF test value $\tau = -16.97750$, which is more negative than even the 1% significance level (-3.456514), and value of "p" is 0.000 which less than significance level of 0.05 as shown in Table 2, so we reject the null hypothesis of unit root test in other words the run start time since midnight of D_2O set of observations I, is a stationary series.

Similarly for set of observations II, the ADF test value $\tau = -13.69809$, which is more negative than even the 1% significance level (-3.456514), and value of "p" is 0.000 which less than significance level of 0.05 as shown in the Table 2, so we can reject the null of non-stationary data, in other words the run start time since midnight of salt water set of observations II, is a stationary series.

AIC, SC and coefficient of determination (R^2) results for the trend detection for both set of observations I & II are tabulated in Table 1.

Illustration on Table 2. Largest Periodogram peaks of Time of start of run since midnight of SNO D_2O set of observations.

Figure 9, Plot of Periodogram VS Frequency of Time of start of run since midnight of the neutrino interaction at SNO for heavy water set of observations.



Errors (SEs) are white noise estimates.

Hamming weights ranges from 0.035714 to 0.446429.

The Figure 9 sketched the magnitude of periodogram vs. frequency of our 2nd set of observations, Time of start of run since midnight of SNO salt water sample taken containing 250 entries from 2 Nov 2001 to 08 Apr 2002.

Five leading peak values are listed in the Table 2. It can be observed that there are various (ACFs,) peaks but largest peak of frequency 0.1160 about 6.19×10^9 is one of the two values exceeding the range of 6×10^9 as represented in the Figure 9.

Furthermore, beside periodogram exhibits numerous peaks of larger and smaller values.

Periodogram Manifestation

Seventeen major peaks values ranging above 3 × 109 are tabulated in Table 3 these peaks are also ranked shown in column 6.

Out of these three next major sequential higher peaks are 6.180 × 109, 5.324 × 109 and 5.12 × 109 at frequencies 0.320, 0.3480 and 0.1000 respectively.

The fluctuating nature of Periodogram values exhibit various moderate and lowest peaks at range of frequencies.

The Periodogram against periods of said set of observations II exemplified in Figure 4 indicates largest peaks of 6.19 × 109 at period 8.6207.

Periodogram having numerous peaks of larger and smaller values jumbled up within the period of almost 10 values at the range of 250.

Out of the seventeen aforesaid major peaks values tabulated in 2, next highest three major peaks are at periods 3.1250, 2.8736 and 10.0000 respectively.

Figure 10, Depiction of Periodogram VS Frequency of time of start of run since midnight of the neutrino interaction at SNO salt water set of observations.

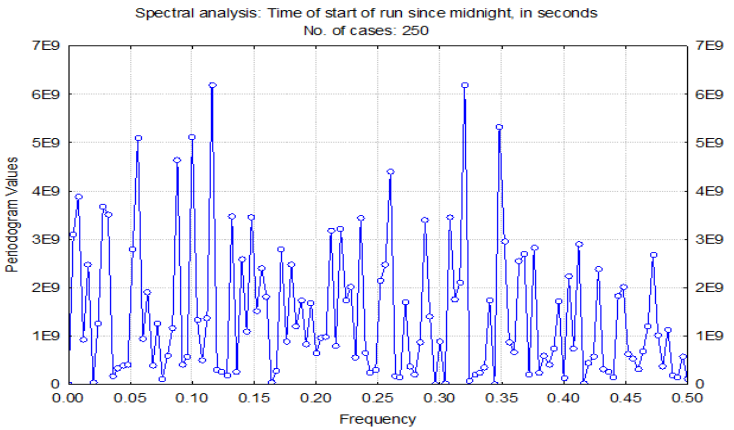
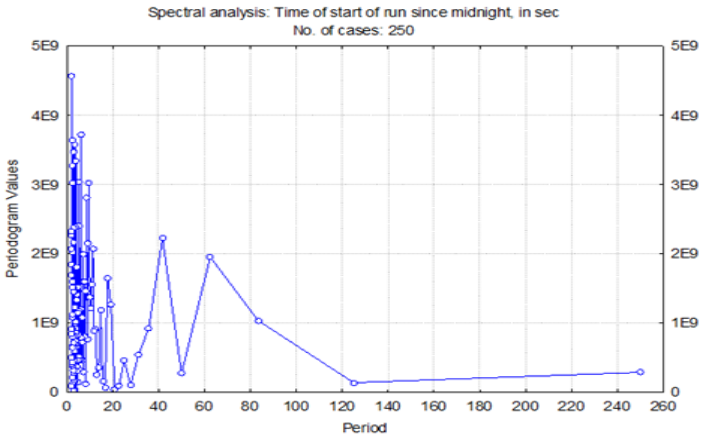


Figure 11, Temporal behavior of Periodogram for the time of start of run since midnight of the neutrino interaction at SNO D_2O set of observations.



Explanation on the periodogram Figures (11,12 13 and 14).

The Periodogram vs. Periods of same set of observations I "Time of start of run since midnight" illustrated in Figure 10 manifests largest peaks of 4.56×109 at Period 2.0492.

Periodogram having numerous peaks of larger and smaller values jumbled up within the period of 10, whereas the range is 250 entries.

Out of aforesaid major peaks, other ten values are tabulated in Table 3 indicates corresponding periods also next three successive highest major peaks are at periods of 6.0976, 2.6042 and 3.4722 respectively.

The Table 3 exhibits a comparative study of the values obtained from Periodogram, ACF, PACF frequencies, autocorrelation and partial coefficient values for AR(1) models SNO D_2O set of observations (Time of start of run since mid-night, in Sec), $N = 250$.

Figure 12, Log period Vs Periodogram illustration of the time of start of run since midnight of the neutrino interaction at SNO D_2O set of observations.

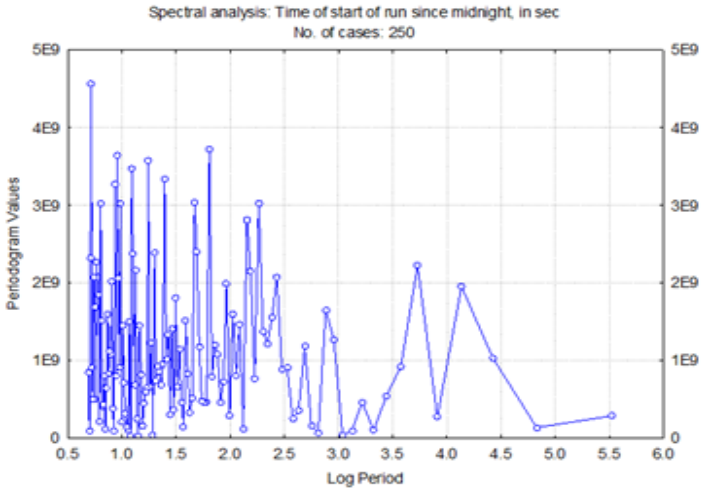


Figure 13, Inspection of Periodogram verses Log period of time of start of run since midnight of the neutrino interaction at SNO Salt water set of observations.

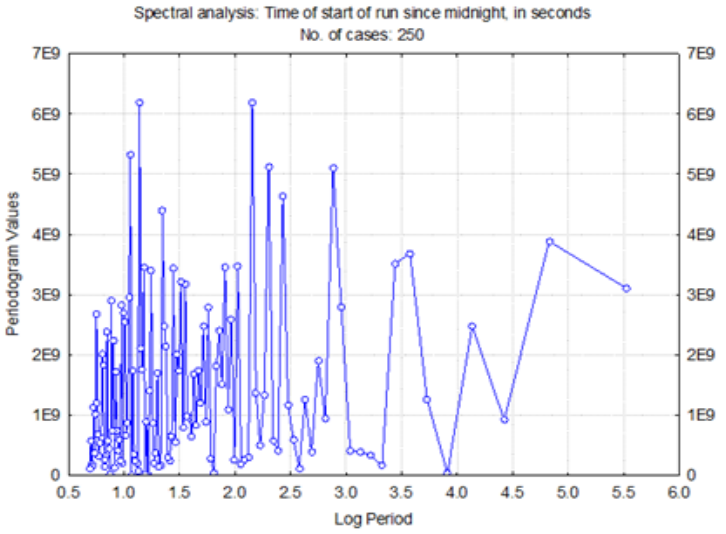
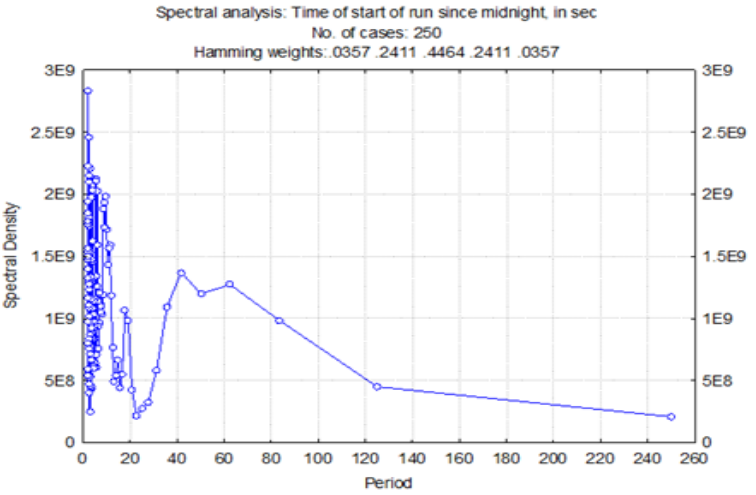


Figure 14, Depiction of Spectral Density verses Period of time of start of run since midnight of the neutrino interaction at SNO D_2O set of observations.



COMMENTS AND CONCLUSION

While analyzing the graphs and values given in the Haar wavelets, it has been concluded that comparatively, the Time of start of run since midnight of SNO salt water set of observations level 1-8 depict the start of neutrino interaction with salt water set of observations.

In the mean status, the Time of start of run since midnight of SNO salt water set of observations possess denoising representation of (Fixed Form threshold, Unscaled white noise) at level – 2 for Haar wavelet family for SNO 1st variable Run start Time since midnight in Sec of D_2O data indicating original and de-noised signal, Original coefficient, Threshold coefficient and Original detailed coefficients of level

Graphical presentations of Coefficient of Detail signal, histogram and cumulative histogram for Haar wavelet family at level – 1 for SNO 1st variable Run start Time since midnight in Sec of D_2O data.

Also, Histogram and Cumulative Histogram of Residuals (Haar, Level-2) of Denoised SNO 1st variable Run start Time since midnight in Sec by Soft Thresholding Method (Mini max Threshold) in Scaled White Noise of D_2O data.

Haar wavelets manifest the hidden periodicities in the neutrino emission. The presentation depicts the different levels from coarse to fine.

If we take graphs and values of the time domain and frequency domain then it could be concluded that the Time of start of run since midnight of SNO salt water set of observations has highest Periodogram peaks of 6.91 × 109 for SNO D_2O set of observations having highest peak of 4.56 × 109. These depict the start of neutrino interaction is strong in salt water set of observations.

Similarly, the Time of start of run since midnight of SNO salt water set of observations posses highest Spectral density peaks of 3.3548 × 109 in comparison to SNO D_2O set of

observations having highest peak of 2.8378 × 109 that depicts neutrino interaction has higher Spectral density value in salt water set of observations.

The variable analyzed in this study is "Time of start of run since midnight" means the value so recorded is the time in Sec I, start time of neutrino interaction.

Higher peaks of Spectral density and periodogram indicates delayed emission of neutrino interaction; the set of observations discussed in this study, are recorded at the same geographical position, in the same laboratory, in almost same environment, the time frame as mentioned above is carefully selected to be of the same date and month despite different years for better similarity.

Beside the time frame, the difference is medium, i.e. heavy water (D_2O) and Salt water.

It has been established that neutrinos existence and emissions from deep space have been detected in world laboratories as mentioned in the above sections.

These laboratories are recording regularly neutrinos set of observations to understand behavior and characteristic of the same.

The samples of these observations recorded in the Sudbury Neutrino Observatory (SNO), Ontario, Canada, from Nov 1999 to May 2001 of heavy water (D_2O) and set-II from Jul 2001 to Aug 2003 of salt water with Run start Time since midnight are analyzed here to examine time-based variations using standard software to construct HAAR presentation of different types and family of wavelets.

Frequency illumination, Periodogram and along some Spectral density Curvatures that are exhibited both in time and frequency domains.

No sign of time base variation has been manifested in both set of observations in joint analyses of the two sets of observations as it has been illustrated by One of SNO papers during the year 2005 (40).

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I wish to endorse my sincere thanks to Canadian Laboratory known as Sudbury Neutrino Observatory (SNO) for providing the set of observations to undertake this kind of research work in Pakistan where there is no neutrino detection laboratory does exist.

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EXPLORATION OF FRACTAL ATTITUDE TO DESIGNATE THE CONVOLUTION OF NEUTRINO FLUX REACHING FROM DEEP

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ABSTRACT

It has been known from the start of 20th century that universe is composed of atoms, electrons and protons being integral part of it. The neutrino was discovered during 20th century revealed as an amazement. The observations of β decay in the 1920's, led to discovery of neutrino. The name of neutrino was derived from the Italian word Neutrino by Enrico Fermi. Initially neutrino was evaluated as weigh less, no interaction with other particles. Neutrino is a fundamental particle, found as leptons in the universe.

In this communication, we intend to evaluate the universal behavior of neutrino interaction with the other matter. This interaction seems to be complex and intricate because the emission of neutrino starts from deep space to reach our Biosphere. This journey from outer space to our earth is highly nonlinear. Therefore, we have to investigate approaches to assess the complexity in the nature and adopted the Fractal dimension of the neutrinos. Historically the word fractal originated from fractus, a Latin word, that was first used by Mandelbrot during the year 1975.

Moreover, in this communication, our aim and objective are to investigate the neutrinos affairs to introduce a new inflexion We have opened a fresh field of analyzing, and modeling different aspects of neutrinos using Fractal dimension approach with its ingredients. We intended to carry out research at the Institute of Space Science and Technology, University of Karachi with the collaboration of the Imam Ahmed Raza University of Emerging Sciences and Technologies. Also International Neutrinos Detection Laboratories Such as Sudbury Neutrino Observatory, Canada, and second well known laboratory is in Japan collaborated this research in Pakistan by providing set of observations.

This word indicates to describe the irregular shapes of naturally occurring phenomena (Mandelbrot, 1985; Turner et al., 1998). We should also remembered that the formal manifestation of this approach during the year 1982 "the measure of space filling ability of a curve" (Ostwald and Vaughan, 2016). This concept of fractality has been utilized in describing the multi resolution phenomena occurring in real world. In this paper we have utilized set of observations of neutrino flux obtained from one of the two international

laboratories. (1) Sudbury Neutrino Observatory (SNO) Canada and (2) Kamikando Observatory Japan. Especially from SNO.

In this paper we intend to present salient features of Fractal Dimension, Determination of Fractal-description, Hurst Exponent (H), Diagnostics of fractal approach and Rescaled Range (R/S) Analysis for both the set of observations.

KEYWORDS

Fractal description, deep space emission, Neutrino Flux, SNO detection station, Manifestation of Fractal dimension.

INTRODUCTION

It has been known for a long time that the basic structure of atom according to Rutherford comprises a nucleus having proton and neutron at the center of the atom and electron moving round the nucleus. It has been explored that during the 19th century the origin of neutrino is linked to the discovery of β decay of nuclei. At the same time after Solvay conference the famous Physicist Fermi established theory of beta decay and developed the structure of beta spectrum and the interpretation of rest mass of the neutrino [1-3]. Pauli during the year 1930 affirmed the familiarity of energy and momentum conservation of the process of β-decays via hypothesizing neutrino.

We intend to evaluate the universal behavior of neutrino interaction with the other matter. This interaction seems to be complex and intricate because the emission of neutrino starts from deep space to reach our Biosphere. This journey from outer space to our earth is highly nonlinear. Historically the word fractal originated from fractus, a Latin word, that was first used by Mandelbrot during the year 1975. This word indicates to describe the irregular shapes of naturally occurring phenomena (2-4). We should also remember that the formal manifestation of this approach during the year 1982 “the measure of space filling ability of a curve” [5]. The investigators concluded that fractal geometry is responsible to evaluate the complex nature of the objects under consideration This concept of fractality has been

Fractal dimension technique has been utilized for irregular and complex tissue based on medical images and satellite imageries. These images were not characterized from Euclidean geometry attempted to offer non-Euclidean aspects [6-10].

At this phase of research, concept of fractal is considered very helpful to understand and reveal the behavior of subject phenomena, as it has followed power law behavior in frequency domain. In space phenomena, at times we are to discover complete absence of order in a system. To attempt nonlinear dynamical approach, we find literature covering correlation dimension, nonlinear prediction residuals and Lyapunov exponent [11] BDS etc. that may assist in differentiating chaotic character from the noise. Knowing the capability of these processes to discriminate the intrinsic and physical character of a system from set of available observations, to determine number of variables required to model the physical behavior of the system such as neutrino emission from deep space (Bandt et al., 2010).

2. SOURCE OF OBSERVATIONS

The Sudbury Neutrino Observatory recorded two sets of observations, first starting from 2nd. Nov. 1999 to 27th May 2001 using heavy water (D_2O) as interacting medium, having parameters recorded variables, Run Start Time since midnight and Length of run, in Sec, having 559 entries and second set of observations starting from 26th July 2001 to 28th Aug 2003 using salt water as medium of interactions with same particulars recorded, having 1212 entries are taken.

It is noted that for proper symmetric comparison and analysis, samples are so taken that coincide with solar year time starting from 2nd Nov 1999 for heavy water (D_2O) and 2nd Nov 2001 for salt water set of observations thereby celestial position of our Planet Earth that is orbiting round the Sun in the Galaxy should be as same as possible at the time of recording of observations.

3. PHYSICAL FEATURES OF NEUTRINOS

The report on the measurement of the total flux of active ^8B neutrino at SNO from the solar emission indicates neutrino flavor transformation. The CC and ES reaction rates are consistence with earlier results and with the NC reaction rate under the hypothesis of flavor transformation. It has been found that the ^8B neutrino flux is in agreement of Standard solar model. The abrupt changes on daily basis have been reported in the CC, NC, and ES at SNO with different reaction rates.

It has been known that solar neutrinos have been detected at the Sudbury Neutrino Observatory (SNO) showing the decay of ^8B via the charged current (CC) reaction on deuterium and elastic scattering (ES) of electrons. This CC reaction seemed to be more sensitive in case of ν_e's and the ES reaction has sensitivity to ν_μ's and ν_τ's.

It has been studied that the concept of fractal structure seems to be multilayer that can exhibit self-similar hierarchical features [11-12] end to formulate common shapes of the objects that could illustrate similar copies that are governed by following statements:

All these fluxes (CC, ES and NC) data from the pure heavy water D_2O phase presented and analyzed, were in general derived under the assumption of an energy-independent survival probability of undistorted ^8B energy spectrum. The determination of results at SNO for salt fluxes also showed energy dependence and survival probability of same type of neutrino ν_e. SNO Collaboration based on correlations between salt results and its heavy water results, comments “there are correlations being the same detector, these are not large, since some changes have been made to both the detector”.

4. EXECUTION OF FRACTAL DIMENSION TECHNIQUE ALONG WITH HURTS EXPONENTS AND R/S

We intend to mention the complex nature of the emission that could be resolved using the unique technique for analyzing the signals.

In space phenomena, at times we are to discover complete absence of order in a system. To attempt nonlinear dynamical approach, we find literature covering correlation dimension, nonlinear prediction residuals and Lyapunov exponent [12-14] that may assist

in differentiating chaotic character from the noise. Knowing the capability of these processes to discriminate the intrinsic and physical character of a system from set of available observations, to determine number of variables required to model the physical behavior of the system such as neutrino emission from deep space.

The fractal dimension (D) has the same property. The self-similar structures are simplified form of fractional Brownian motion (FBM) designated by Hurst exponent (H), which illustrated the scaling character of the motion and its value is changes from 0 to 1. Researchers have use and suggest nonlinearity approach in recorded set of observations using R/S technique and investigated the understanding of H computation (Katsev, et al., 2003, Chamoli et al., 2007). The H is used for quantification of fractal scaling in the data analysis (Katsev, et al., 2003; Kale, et al., 2011). For the same reason we have used different methods to compute H and D, firstly Rescaled range analysis, secondly Detrended fluctuation analysis, at third place correlation dimension approach and Lyapunov exponent for referred analysis [14].

5. DETERMINATION OF FRACTAL-DESCRIPTION

It has been studied that the concept of fractal structure seems to be multilayer that can exhibit self-similar hierarchical features [15]. We intend to formulate common shapes of the objects that could illustrate similar copies that are governed by following statements:

$C = 2^D$,
C being number of copies of shape/ object
D being number of dimensions of shape/ object.

If we want to extent the enlargement to more than double, the formula can be further enhanced to:

$C = A^D$
A being the extent to which we want to enlarge
D being dimensions

Note: e.g. for A = 3, we need to multiply each dimension of the fractal by 3.

We can solve the intricate structure of object using the mathematical expressions given below.

$\text{Log C Log C} = \text{Log } (A^D)$
$\text{Log C} = D * (\text{Log A})$
$D = \text{Log C} / \text{Log A}.$ (1)

To understand we consider one side of the Koch Snowflake (the Koch Curve) shown in Fig 3.1, we observe that 4 Curve stuck together make something 3 times as large, so A = 3 and C = 4, using mathematical relation of D in Eq 3.1, dimensional value “D” of the Koch Curve is 1.26185 (Mandelbrot, 1985).

One side of the Koch Snow Flake (the Koch Curve)

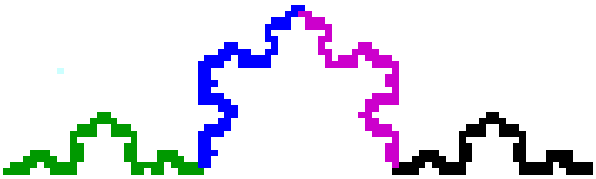

Figure 1: To Understand we consider One Side of the Koch Snowflake (the Koch Curve)

6. FRACTAL DESCRIPTIVE VIEW

The Fractal approach is being used for variety of natural phenomenon showing 'self-similarity' broadly repetitive at randomly smaller and smaller levels (Kale and Butar, 2011). The Fractal Dimension (D) exhibits strong nature of complexity and nonlinear dynamics of natural complex systems (Zai, 1989). A fractal nature with a high convolution is much complex or 'rough' on contrast to a lower dimension and fills much space (Hassan et al., 2014). It will be more convenient to specify the morphological structure of the objects using fractal dimensions. It is to be noted that scaling exponents are given to design regular monofractals, instead of the Hausdorff–Besicovitch dimension [16].

The topological structure for the inequality $1 > D > 2$ does explain that the curve is dimensionally planer and its status is 2-dimensional, some figures are 3-dimensional and illustrating a surface of hypersphere that would be indicated as 3-dimensional. We have seen in above paragraph that the Koch Curve's 1.26285 dimensions greater then 1, so it is formally a fractal. This approach of finding fractal dimension is feasible for self-similar fractals (Mandelbrot, 1985).

In fact, the fractal approach can be considered as an appropriate tool for multiple fields in our real life. This technique for all the dynamical systems that exhibit trajectories in time domain and phase space. The fractal dimension possesses the effective number of degrees of freedom in the dynamical system and thus quantifies its complexity.

7. HURST EXPONENT (H)

We are familiar with the concept of Hurst exponent (H) that are composed of Rescaled range (R/S) and Detrended fluctuation analysis could be regard. Now we are in a position to analyze phenomena of deep space as we have found in present study that estimating long memory or persistence that does illustrate the correctional structure between the observations recorded in various times. If the magnitude of persistence seems to be bigger than the correlation structure larger.

Thus, the Hurst exponent may be utilized as a measure of long-term memory of time series. It does relate to the autocorrelations of the time series and the rate at which these decrease as in the case of lag between pairs of values increases. Hurst exponent is so common that almost all the concerned fields need to involve it as advance techniques of the fractal dimension

The Hurst exponent, H, is defined in terms of the asymptotic behavior of the rescaled range as a function of the time span of a time series as follows:

$$E\left[\frac{R(n)}{S(n)}\right] = Cn^H \quad (2)$$

where, $R(s)$ is the range of the first cumulative deviations from the mean, and is their standard deviation. S(n) is the expected value, is the time span of the observation (number of data points in a time series) and is a constant. System dynamical properties of various Hurst exponents are shown in Table 3.1 [17].

The relation between the Hurst exponent (H) and the fractal dimension (D) is simply $D = 2 - H$, where 2 = Euclidean Dimension (Peters, 1994). For the various natural phenomena with H being about 0.72, the fractal dimension D is about 1.28.

The Hurst exponent is used as a tool to evaluate the relationship between long-term neutrino interaction and time variables associated. The H-value of the series provided a means to evaluate the strength or persistence of the neutrino interaction in the medium i.e. D_2O or salt water, as the set of observations analyzed in this study is recorded in different medium. To formally frame the analysis described here, the hypothesis is that time of start run or length of run will have higher H-values reflective of more persistent neutrino interaction

8. RESCALED RANGE (R/S) ANALYSIS

The integral part of fractal description of time events of the phenomenon from deep space is Rescaled range R/S approach. Various investigators pointed out significance of this technique to categories and highlight persistency such as in 1968 by Mandelbrot and Van Ness [17], studied the concept of fractal analysis, Hurst exponent on time series data and Weng et al. (2008) studied tropical ozone dynamics (Jan, et al., 2018). Hurst developed the method of rescaled range (R/S) Peters, 1994; Peters, 1996).

We have computed the parameters to illustrate the intricacy of neutrino flux by applying the R/S approach. In this case one of the very important parameters namely the Hurst exponent (H) that has been determined by AR (1) values of observations namely time of start of run since midnight, in Sec and length of start of run, in Sec (about 559 data values of D_2O and 1212 values of salt water data) basis by ignoring trend behavior. The standard software has been utilized to explore the parametric values. Moreover, in this investigation, we have estimated parameters of rescaled range approach for three different samples using AR (1) values of time of start of run since midnight, in Sec records from 2 Nov 1999 to 27 May 2001 of D_2O and both variables time of start of run since midnight, in Sec and length of start of run, in Sec records from 26 Jul 2001 to 28 Aug 2003 for salt water

R/S approach - SNO D_2O 1st variable

We have calculated the parametric values using (R/S) technique for Time of start of run since midnight (Neutrino interaction) of SNO D_2O (HEAVY water) data sample taken comprising 559 entries from 2 Nov 1999 to 27 May 2001

Sample: 1 Range (R) = 86210, Std. (S) = 25725,

Data size; N = 512, R/S = 3.351214,

log(N) = 9, log (R/S) = 1.744683.

Sample: 2 Range (R) = 86210, Std. (S) = 23977,

Data size; N = 256, R/S = 3.595529,

log(N) = 8, log (R/S) = 1.846204.

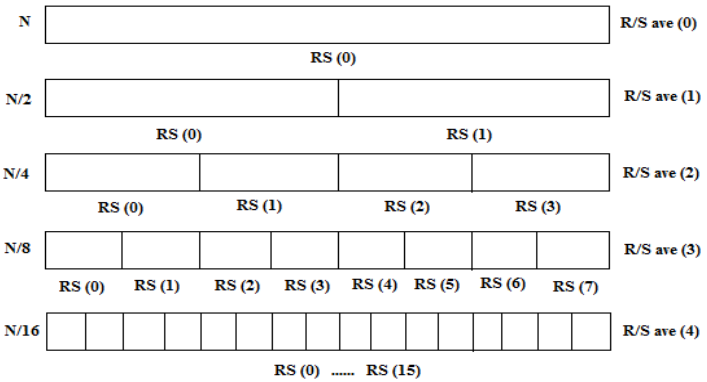

Figure 2: Dividing a Time Series and Sampling Procedure for R/s Analysis and DF

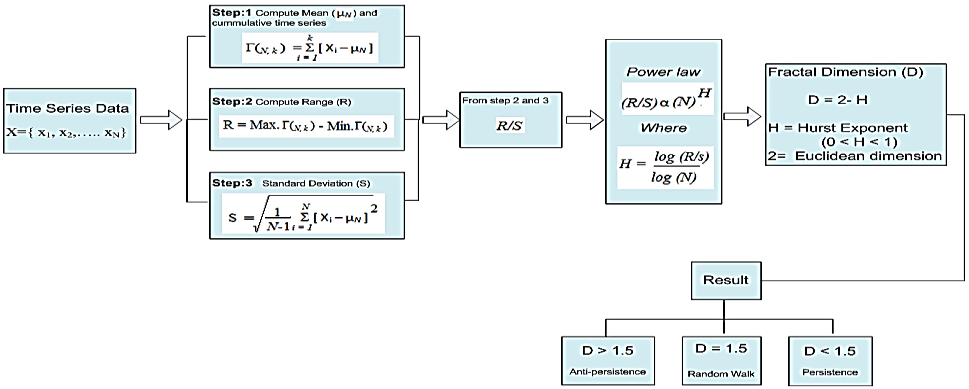

Figure 3: Flow Chart for Hurst Exponent and Fractal Dimension Procedure

9. DE TRENDED FLUCTUATION ANALYSIS (DFA)

We have to decide that the Detrended fluctuation analysis (DFA), a method that determine the statistical self-affinity of the phenomena in stochastic processes, chaos theory and time series analysis. DFA is a useful tool for analyzing time series, appear to have long-memory processes e.g. power-law, decaying autocorrelation function etc. The obtained exponent is similar to Hurst exponent. Additionally, DFA may also be applied to signals whose underlying statistics/ dynamics i.e. mean and variance are non-stationary (changing with time). It is related to measures based upon spectral techniques such as autocorrelation and Fourier transform

The multifractal characterization of nonstationary time series is based on a generalization of the detrended fluctuation analysis (DFA). Both Multifractal DFA method and standard partition function-based multifractal formalism approaches are equivalent for stationary signals with compact support. Multifractal DFA method can reliably determine the multifractal scaling behavior of time series [17]. It has been pointed out by the research workers that Detrended Fluctuation Analysis (DFA) has been established as an important tool for the detection of long-range autocorrelations in particular time events.

The outcome of detrended fluctuation analysis (DFA) is a scaling analysis method used to quantify long-range power-law correlations in signals. Many physical and biological signals are “noisy”, heterogeneous and exhibit different types of non-stationarities, which can affect the correlation properties of these signals

There is an extension in R/S estimation in order to diagnose the fractal characters. It was applied to examine the long-range power-law correlations properties (Uma and Selvaraj, 2012). Many researchers studied DFA in different areas like physics, engineering, and finance, medical as well as meteorology to investigate scaling exponent (Hurst exponent). Matsoukas applied different Detrending approaches to observe fluctuation in hydrological data (Matsoukas et al. 2000). DFA was introduced first time to evaluate the correlation properties of signals

10. CORRELATION DIMENSION (CD)

Correlation Dimension (CD) may be considered as a measure for the number of independent variables needed to define the total system, any system generating the RR interval (the time elapsed between two successive R-waves of complex signal) time series, in phase space. Nonlinear dynamic properties in a time-based system can be detected with correlation dimension (CD). While analyzing any set of observations of a nonlinear systems, it is unfortunate that a time series can be measured for a single system variable. To explain, in the SNO set of observations two variables “TOSRSM and LOR” are recorded in time frame but independently. It was only practicable to measure the fluctuations in one variable with respect to time [18-19].

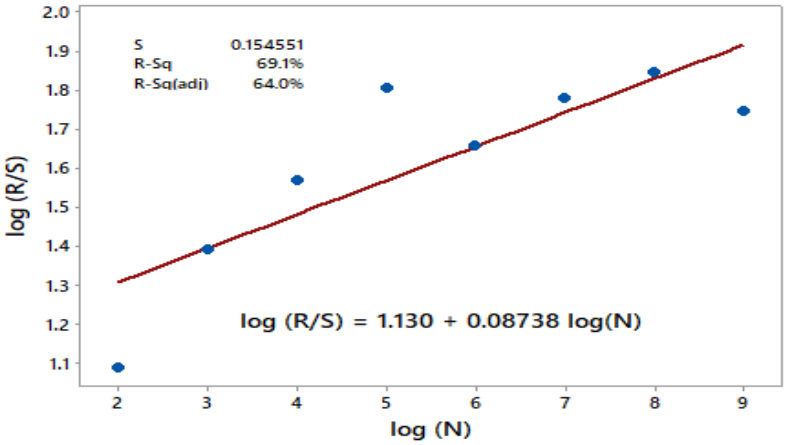

Figure 4: Sketch of (R/S) – SNO Time of Start of Run since Midnight of D_2O Sample

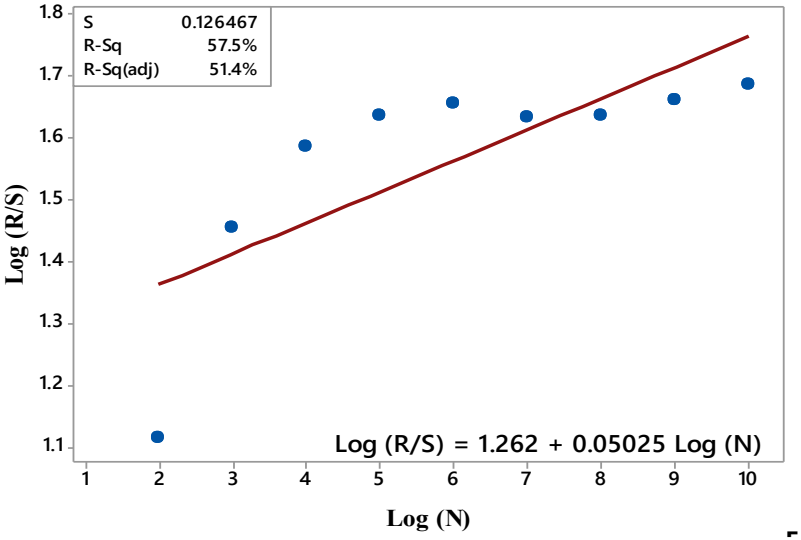

Figure 5: (R/S) - SNO Time of Start of Run since Midnight of Salt water sample

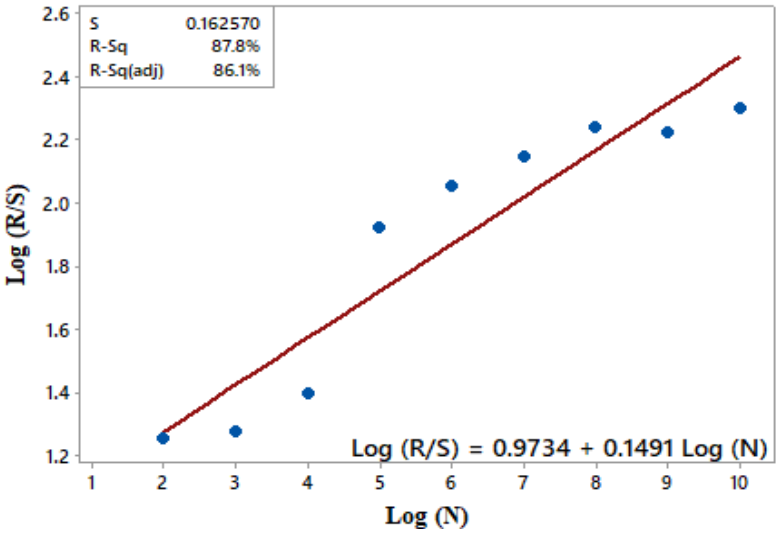

Figure 6: (R/S) – SNO Length of Run, in Sec of Salt Water Sample

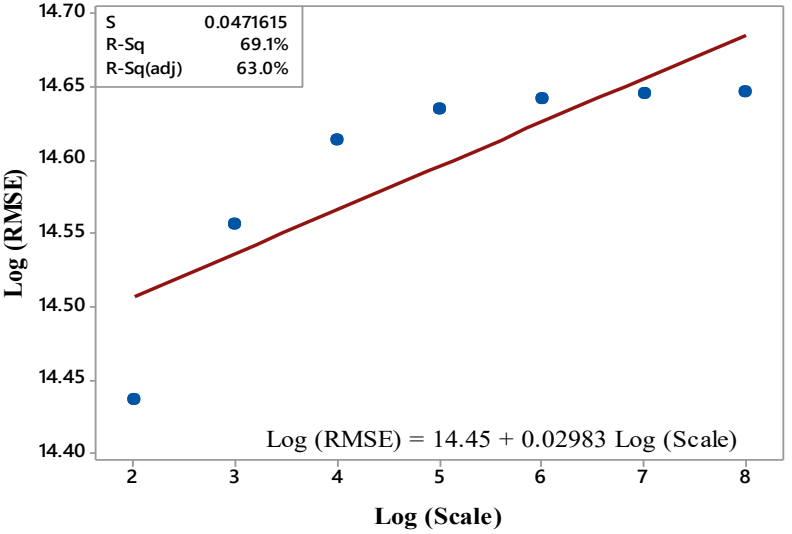

Figure 7: DFA, Time of Start of Run Since Midnight of SNO D_2O Sample

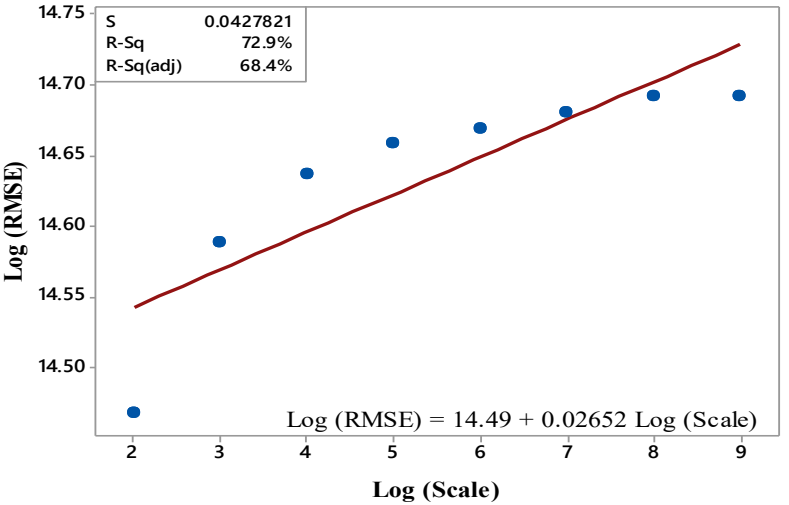

Figure 8: DFA, Time of Start of Run Since Midnight of SNO Salt Water Sample

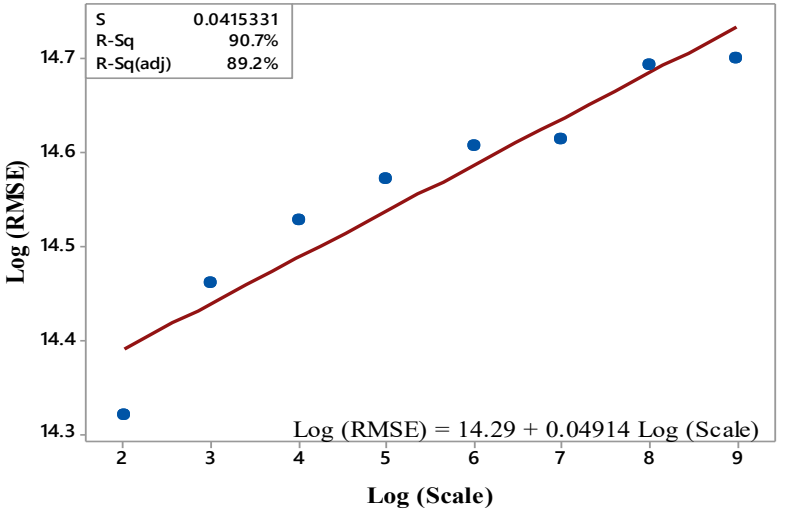

Figure 9: DFA, Length of Run in Sec, of SNO Salt Water Data Sample

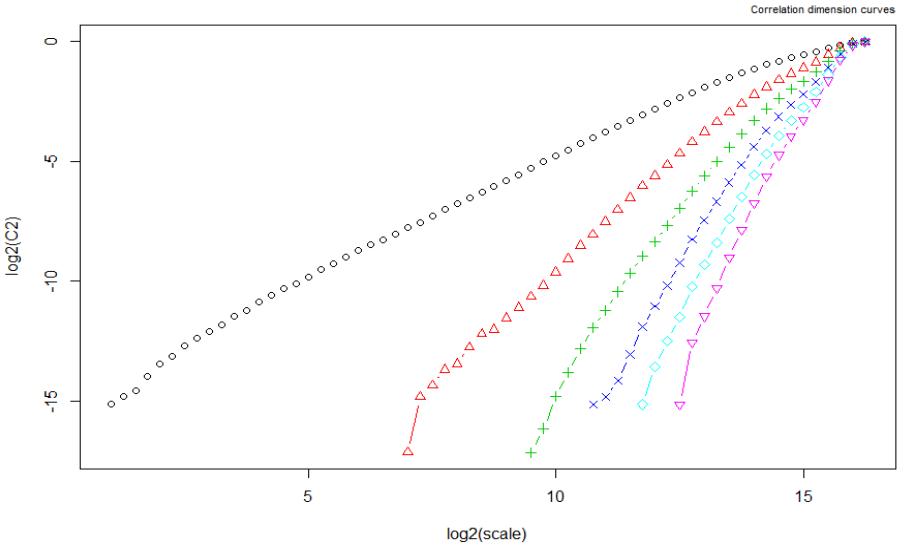

Figure 10: CD Plot of Correlation Function of Embedding Dimensions (E), for TOSR, in sec of SNO D_2O Sample

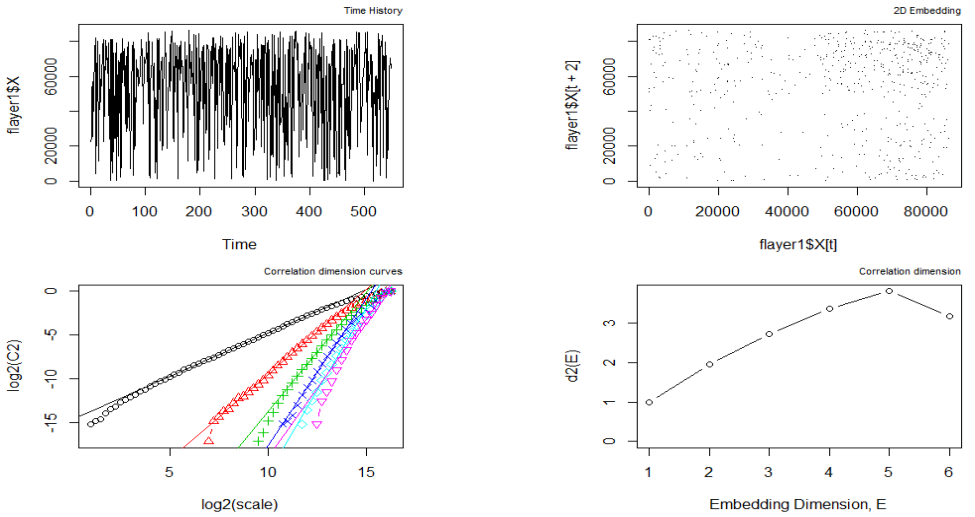

Figure 11: Correlation b/w Correlation Dimension (slope = CD) and Embedding Dimension (E), for TOSR, in sec of SNO D_2O Sample

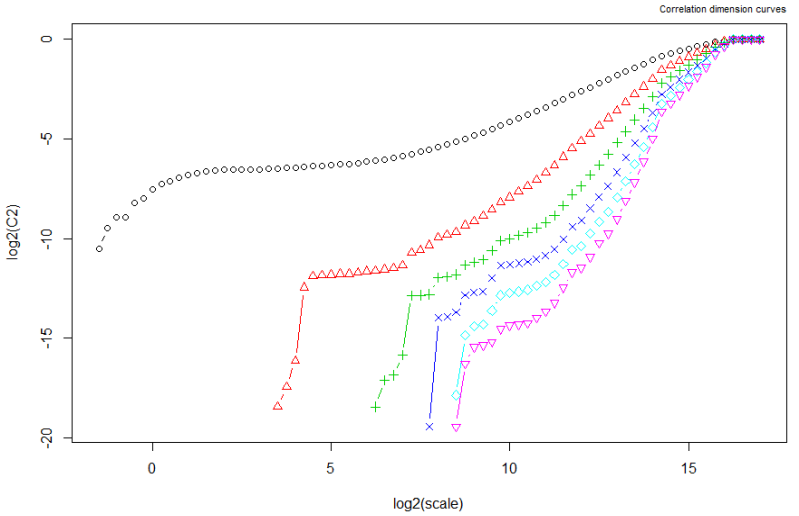

Figure 12: Display of Correlation Dimension of Correlation Function of Embedding Dimensions (E), for LOR, in sec of SNO Salt Water Sample

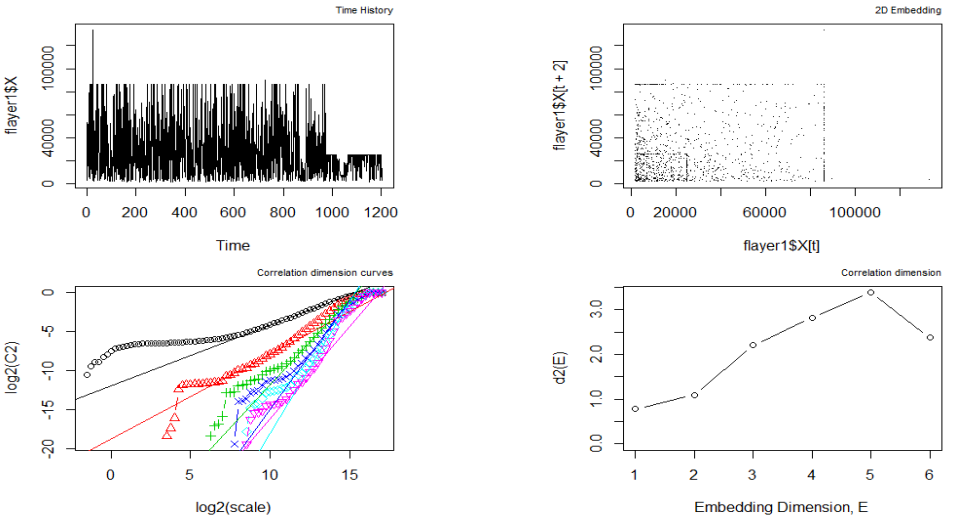

Figure 13: Real Fashion of Correlation b/w Correlation Dimension (slope = CD) and Embedding Dimension (E), for LOR, in sec of SNO Salt Water Sample

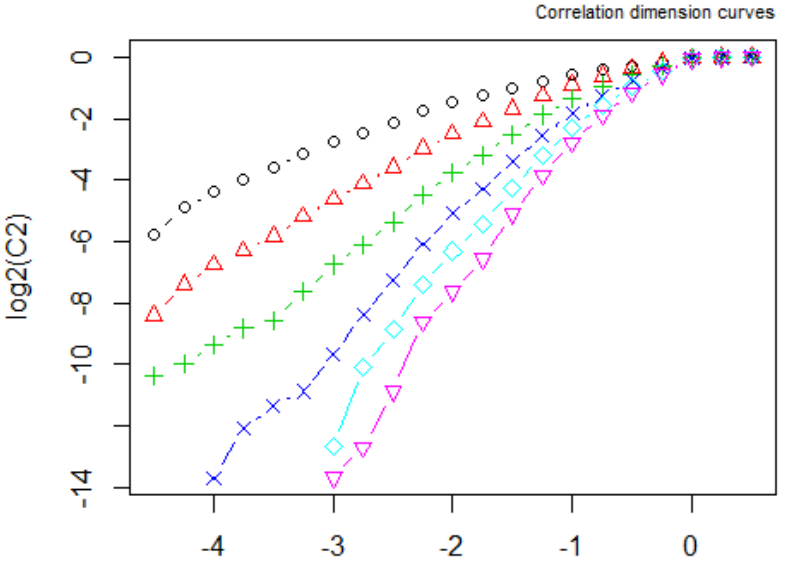

Figure 14: Depiction of Correlation Dimension of Correlation Function of Embedding Dimensions (E), for Mean 8B Solar Neutrinos Flux against Live Time (JST) of 10 Days Bin Data Sample

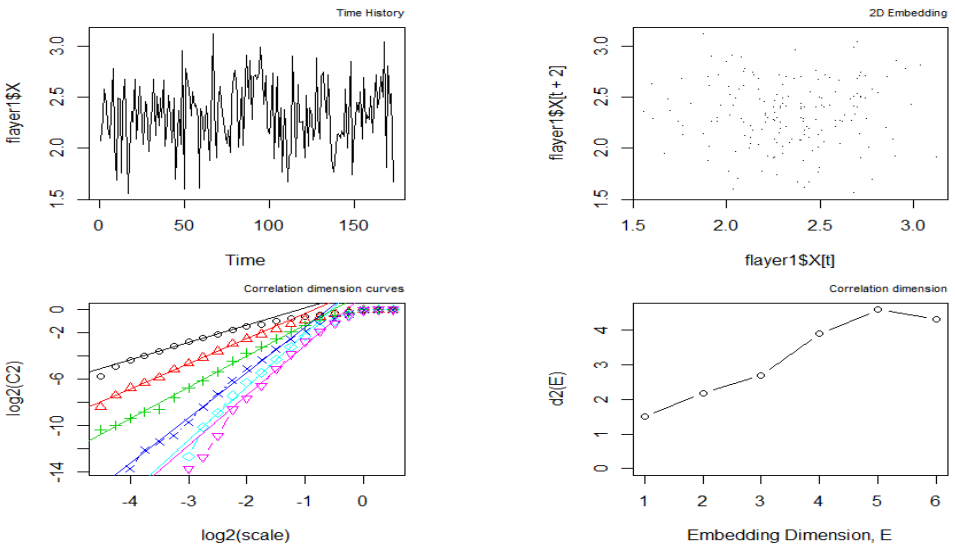

Figure 15: Representation of Correlation b/w Correlation Dimension (Slope = CD) and Embedding Dimension (E), for Mean 8B Solar Neutrinos Flux against Live Time (JST) of 10 Days Bin Data Sample

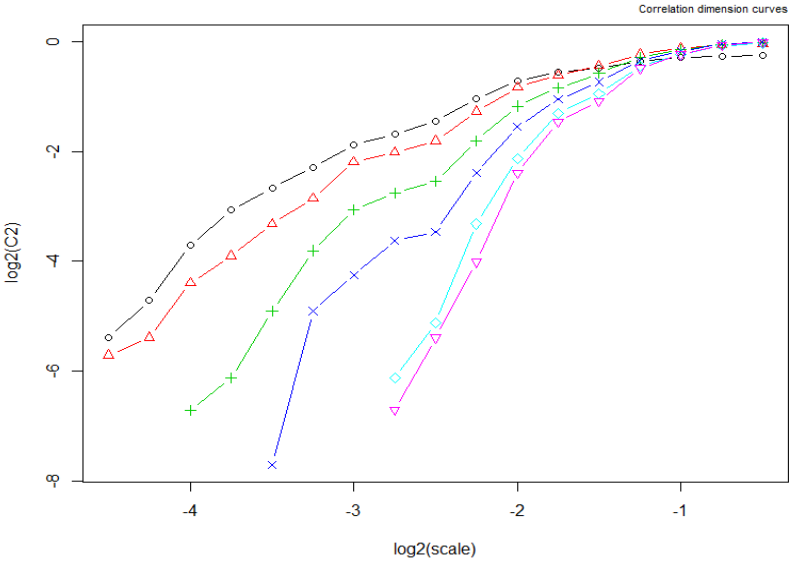

Figure 16: Representation of Correlation Dimension Plot of Correlation Function of Embedding Dimensions (E), for Mean 8B Solar Neutrinos Flux against Live Time (JST) of 45 days Bin Data Sample

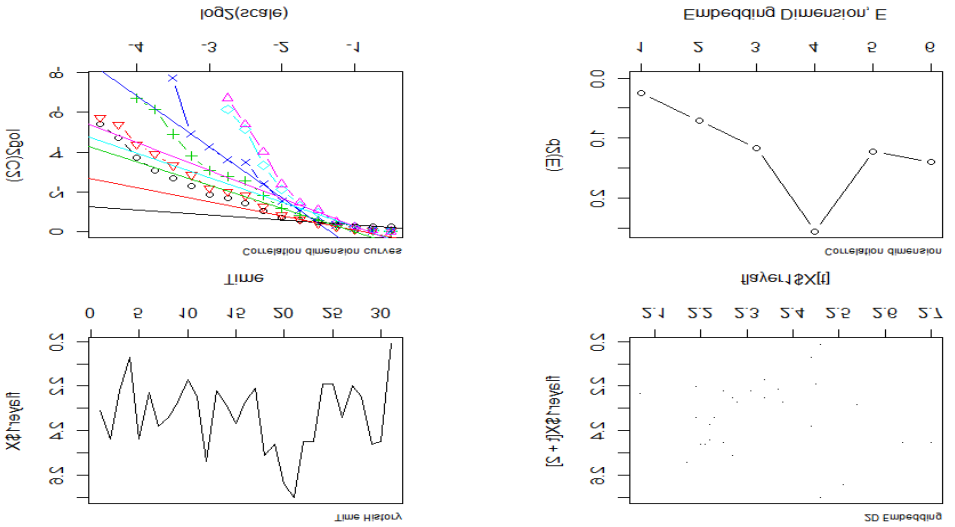

Figure 17: Solar Neutrinos Flux Manifestation of Correlation b/w Correlation Dimension (Slope = CD) and Embedding Dimension (E), for Mean 8B against Live Time (JST) of 45 Days Bin Data Sample

11. COMMENTS AND CONCLUSION

While analyzing the graphs and values, it has been concluded that comparatively, the Time of start of run since midnight of SNO salt water set of observations has highest. Similarly, the Time of start of run since midnight of SNO salt water set of observations possess highest. The fractality, rescaled range analyses with Correlation b/w Correlation Dimension (slope = CD) and Embedding Dimension (E), for Mean 8B against Live Time (JST) of 45 days bin data sample.

The variable analyzed in this study is "Time of start of run since midnight" means the value so recorded is the time in Sec I, start time of neutrino interaction. The set of observations discussed in this study, are recorded at the same geographical position, in the same laboratory, in almost same environment, the time frame as mentioned above is carefully selected to be of the same date and month despite different years for better similarity. Beside the time frame, the difference is medium, i.e. heavy water (D_2O) and Salt water.

It has been established that neutrinos existence and emissions from deep space have been detected in world laboratories as mentioned in the above sections. These laboratories are recording regularly neutrinos set of observations to understand behavior and characteristic of the same. The samples of these observations recorded in the Sudbury Neutrino Observatory (SNO), Ontario, Canada, from Nov 1999 to May 2001 of heavy water (D_2O) and set-II from Jul 2001 to Aug 2003 of salt water with Run start Time since midnight are analyzed here to examine time based variations using standard software to construct No sign of time base variation has been manifested in set of observations in joint analyses of the two sets of observations as it has been illustrated by One of SNO papers during the year 2005 [37) research work is part of my Ph.D. dissertation. From it has been observed that the intricacy of the signals of neutrino flux has been manifested and deep space emissions complexity is conformed.

ACKNOWLEDGEMENT

First of all, I would like to pay my thanks to organizers of this conference which is being held under the au species of Islamic Society; of Statistical Sciences (ISOSS) and University of Sargodha. They invited me as an Invited Speaker.

22nd ISOSS:

I wish to endorse my sincere thanks to Canadian laboratory known as Sudbury Neutrino Observatory (SNO) for providing the set of observations to undertake this kind of research work in Pakistan where there is no neutrino detection laboratory does exist. Moreover, I wish that I may be obliged to enjoy the support of University of Karachi and Institute of Space Science and Technology. This research work is part of my Ph.D. student's dissertation.

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OPTIMAL FORECASTING THROUGH DYNAMIC LINEAR MODEL SYSTEM

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ABSTRACT

Accurate and timely forecasting plays a crucial role in informed decision making across real world applications. However, empirical time series data are often contaminated by colored noise processes which undermine the performance of conventional statistical models and lead to suboptimal forecasts. This study highlights the importance of identifying the underlying structure of such noise and demonstrates the use of Dynamic Linear Models (DLMs) to effectively filter it through a purpose-built software package that employs Kalman Filter algorithm within a Bayesian framework.

A novel DLM formulation is introduced, incorporating the product of an autoregressive coefficient and an appropriately chosen damping factor, superimposed in the Transition Matrix and enabling online learning of unknown variances. Furthermore, the Akram Test Statistic (ATS) is utilized to identify the most appropriate model and assess its capacity to produce superior forecasts. The proposed approach ensures residual whiteness and maintains model parsimony, ultimately yielding forecasts with enhanced statistical accuracy and reliability.

1. INTRODUCTION

Forecasting is indispensable for strategic planning, operational optimization, and risk mitigation across a wide range of real-life systems. Conventional forecasting methodologies are typically predicated on the assumption of independent, identically distributed residual disturbances. However, real world data rarely satisfy this assumption due to a plethora of reasons because measurements are affected by colored noise, which includes correlated errors from different sources. When colored noise is present, static model formulations become structurally inadequate: parameter estimates become biased, diagnostic tests fail, and predictive intervals lose interpretability. These limitations ask for the use of state space approaches that explicitly allow parameters to evolve over time while dynamically incorporating uncertainty. Dynamic Linear Models (DLMs), also known as state space methods, originally developed in control engineering for purposes such as tracking rockets, space navigation etc., are quite fundamental to dynamic modelling and provide an elegant framework within which time series models can be developed and fitted iteratively to offer a natural solution to this challenge(Kalman, 1960; Kalman and Bucy, 1961; Harrison and Stevenson, 1976; Pole et. al., 1999). Their sequential updating and filtering capabilities allow them to adjust to evolving noise patterns and parameter

uncertainty. This study introduces a simple yet effective novel DLM formulation well suited for noisy real-life data and designed to filter colored noise through an explicitly constructed transition matrix plus a rigorous model identification procedure to achieve optimal forecasting performance

Choosing the model attaining the optimal generalization is very crucial in any statistical data analysis. Comprehensive surveys on model selection are available in (Linhart and Zuchini, 1986; Shibata, 1989; Shao, 1997; Burnham and Anderson, 2002). The two most widely used standard criteria for this purpose include Akaike's (Akaike, 1973) and Bayesian (Schwarz, 1978) information criteria. Another very simple and non-parametric test statistic for the identification of statistical models for analysis and forecasting purposes is Akram test statistic (ATS) proposed by (Akram, 2001). Besides various model selection criteria, different forecast accuracy measures like bias, mean absolute error (MAE), mean absolute percentage error (MAPE), mean square error (MSE), root mean square error (RMSE), tracking signal (TS), whiteness of residuals, and Durbin-Watson test statistic (Durbin and Watson, 1951) are also available to evaluate the accuracy of forecasts.

An effort has been made to bring excellence in forecasting affairs in a very simple way by employing an especially developed software package written in C++ language and based on the Kalman filter algorithm within a Bayesian framework. The software package generates dynamic forecasts besides computation of different forecast accuracy measures. Furthermore, a novel model which has not been used earlier in statistical literature is applied. In this model, we have superimposed the product of autoregressive (AR) coefficient and a suitable valued damping factor in the transition matrix G ensuring that the first element of G matrix for colored noise processes does not become one so as to keep the system stable. The program is capable of not only dealing with white noise processes but also colored noise processes. After the detection of colored noise process, the model is restructured to filter the color of noise process.

The paper is organized as under, Section 2 describes Akaike's (AIC) and Bayesian information criterion (BIC) plus Akram test statistic (ATS). Bayesian forecasting is discussed in section 3. An example based on real life data along with summarization of results and conclusions are put in the last section.

2. MODEL SELECTION

Effective decision making largely depends on identifying the best model amongst a collection of several viable candidate models to characterize the sample data properly. The model identification approaches have their own merits and demerits. Most of them depend on a host of factors for their applications to real life data sets. A brief introduction of AIC, BIC and ATS is presented below.

2.1 Akaike's Information Criterion

Professor Hirotugu Akaike utilized the Kullback-Leibler distance as a fundamental basis for the model selection criterion to propose AIC (Akaike, 1973). Having solid foundation in information theory, it is given by

$$AIC = n\,(1 + ln(2\pi)) - 2\,ln\,(L(\theta\hat{}\mid data)) + 2K$$
$$= n\,(1 + ln(2\pi)) + n\,ln\,\sigma 2 + 2K$$

or equivalently

$$= n\,ln\,(RSS/n)\; +\; 2\,K$$

where ln $(L(\theta\hat{}\mid data))$ is the maximized log-likelihood over the unknown model parameters (θ) given the data, ' ln' implies natural or Naperian logarithm, RSS means the residual sum of squares (also called error sum of squares, SSE), K denotes the number of estimable parameters in the model, and n indicates the number of observations. The aim of AIC is to penalize the large value of K, that is, the number of free parameters so as to ensure a parsimonious model. Thus, AIC measures the goodness of fit between the fitted approximating model and the generating or true model. A model minimizing the AIC is considered to be the most appropriate one.

2.2 Bayesian Information Criterion

BIC was derived from much older notion of Bayesian statistics by Schwarz (1978). It chooses that model which maximizes the probability of describing a data set and is constructed by same a priori information. Mathematically,

$$BIC = -2\,L(\theta\hat{}) + K\,ln\,n$$

The notations have the same meanings as in case of AIC. A model having a smaller BIC value is preferred.

2.3 Akram Test Statistic

ATS is a non-parametric test statistic based on average string length (ASL) which is the mean of troughs and peaks of forecast errors yielded by the model. It is applied to check the suitability and capability of a model to generate optimum forecasts. To search a suitable model by applying ATS, one has to proceed systematically as highlighted by (Akram and Ahmed, 2007; Ahmed and Akram, 2007).

Analyse discrete time series using a candidate model and estimate the parameters of the model using an optimum estimation technique. Generate one-step-ahead forecasts and residuals. Standardize the residuals and then compute $ASL = (ne - 1)/(n - \rightarrow n + + n + \rightarrow n -)$ where ne is the number of residuals, $n - \rightarrow n +$ represents the number of shifts from negative to positive signs of the standardized residulas, while $n + \rightarrow n -$ means the reverse meaning. Formulate the null and alternative hypotheses and compute the test statistic τ with the constraint $n \geq 30$ as

$$ATS\colon \tau = \{\,2\,(n_e - 1)/(n - \rightarrow n + + n + \rightarrow n -)\}$$

The notations have the usual meanings as described above.

For making decision on the fate of null hypothesis, define the acceptance and rejection regions at some level of significance. Accept H_0 if the value of test statistic τ lies in the acceptance region; otherwise, reject H_0 and accept H_1. The acceptance of H_0 implies that the residuals are white; whereas rejection of H_0 leads us to the conclusion that the residuals are not white, but colored. In case of colored noise process, the question is, of what type this noise is? Autoregressive (AR) type, moving average (MA) or a mixture of the two i.e., ARMA type and what is the order of the noise process? p, q or (p, q). Here, discussion is

confined to $AR(p)$ processes because an ARMA process can be approximated and adequately represented by an AR process (Anděl, 1981).

If H_0 is rejected again, it means that the model with AR(1) colored noise component, has failed to filter the noise and therefore is not suitable for analysis of data. In this case, determine Φ again from the table of theoretical values of ASL (Akram, 2001). The candidate model with AR(1) process would therefore be considered inappropriate and model with AR(1) process will be reconstructed considering $AR(2)$ noise process, using $\Phi 1$ (the first Φ)and $\Phi 2$ (the second Φ) coefficients. The above procedure is repeated until the color is filtered out i.e., H_0 is accepted.

3. METHODOLOGY

3.1 Dynamic Linear Model Structure

A Dynamic Linear Model (DLM) is characterized by the quadruple $\{f, G, V, W\}$. The 'f' vector defines how the process is 'observed'; G is an inherent characteristic of the system governing its motion; V is the observation noise variance and W is the distribution noise variance. The standard framework for a Bayesian state-space model in which the scalar time series yt, observed at equally spaced time points $t = 1,2, \ldots$ is modelled as follows:

Observation equation	$y_t = f\,\theta_t + v_t$	$v_t \sim N\,[0, V]$
State or Evolution equation	$\theta_t = G\,\theta_{t-1}\,W_t$	$W_t \sim N\,[0, W]$

where v_t is the noise of the observation process, w_t is the noise of the population process and they are assumed to be independent and mutually uncorrelated white noise structures. G is the state evolution or transition matrix; f is a column vector and θt is the state vector and the variable of interest.

Assuming prior	$(\theta_{it-1}\|D_{it-1}) \sim N\,[m_{it-1}\,; C_{it-1}]$
and the posterior	$(\theta_{it-1}\|D_{it}) \sim N\,[m_{it}; C_{it}]$

the Kalman filter algorithm works through the following recurrence equations to give the values of mt and C_t.

System matrix	$R_t = GC_{t-1}\,G' + W_t$
Kalman gain vector	$A_t = R_t f'[\,V + fR_t f'] - 1$
Variance-covariance matrix	$C_t = [\,I - A_t f]\,R_t$
Estimate of the parameter	$\theta_t\, m_t = Gm_{t-1} + A_t e_t$
Forecast function	$F_t(k) = fGm_t$
and one-step ahead forecast error	$e_t = y_t - fGm_{t-1}$

3.2 Novel Transition Matrix Design

In the present study, a novel model which has not been used earlier in statistical literature is applied. We have superimposed the product of AR coefficient and a damping factor in the transition matrix G. The detail is as under:

- AR(0) linear growth model

$$G = \begin{bmatrix} 1 & \mu \\ 0 & \lambda \end{bmatrix}$$

- AR(1) colored noise process

$$\begin{bmatrix} \Phi * \beta^{1/2} & 0 \end{bmatrix}$$

$G_1 = 0\ G$

For exponential growth models $\lambda < 1$ and for linear growth models $\lambda = 1$ while μ & covariances are set equal to zeroes.

3.3 Online Variance Learning System

Effective forecasting requires suitable updating of the covariances within the time series structure. The system variance covariance matrix, diagonal in structure, i.e., $Wt = diag\ (w_1, w_2)$, is continuously updated in the following manner.

$$W_{11} = (1 - \beta)\,(1 + \lambda)\,(\lambda - \beta)^* V_t / \lambda\,\beta$$

$$W_{22} = (1 - \beta)(\lambda - \beta)(1 - \lambda\beta^2)(\lambda^2 - \beta)^* V_t / \lambda\beta^2$$

where λs arethe eigenvalues, V_t is the variance and β is a discount factor such that $0 < \beta < min\ |\lambda^2|$. Online variance learning system duly incorporated in the software has the following algorithm

$$V_t = X_t\ /\ N_t$$

$$X_t = \beta v\ X_{t-1} + (1 - f\ A_t)\ e_t^2$$

$$N_t = \beta v\ X_{t-1} + 1$$

where βv is the parameter variance to be defined initially as a prior and then it is updated continuously towards convergence.

4. COMMENTS AND CONCLUSION

Pakistan stock Exchange has maintained its buoyancy and bullish trends during the past few years, but sudden shifts have also been its part and parcel. Data consisting of 164 observations of closing KSE 100-share index in Pakistani Rupee (PKR) during the period July 01, 2004 to February 24, 2005 are depicted as under:

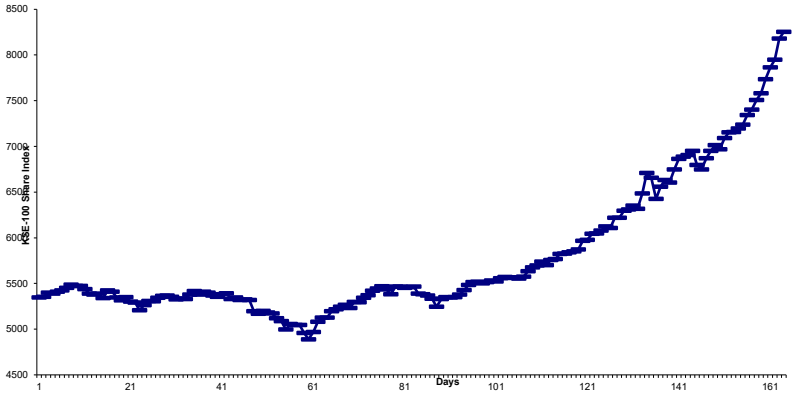

Figure 1: Time Plot for KSE-100 Index

These data are analysed by employing 11 routine statistical models. One-step-ahead forecasts are generated, residuals computed and standardized. Based on these standardized residuals, various statistics of forecast accuracy measuring criteria are computed.

Table 1
Forecast accuracy Measures for KSE 100 Index

Model	Std RSS	AIC	BIC	ASL	D-W	Bias	MAE	MAPE	MSE	RMSE	TS	Sk	Kt
LR	161.00	2.97	12.27	40.75	0.01	2.90e-09	0.84	0.01	1.00	1.00	5.61e-07	0.71	2.96
3-MA	**138.11**	**-24.16**	**-17.96**	**4.40**	**0.62**	**3.63e-08**	**0.66**	**0.01**	**0.85**	**0.92**	**8.89e-06**	**0.49**	**0.34**
2-MAT	160.99	2.96	12.26	1.48	2.74	0.00	0.71	0.01	0.99	0.99	0.00	0.61	0.23
SES	149.18	-11.53	-5.33	2.23	1.63	-8.17e-08	0.69	0.01	0.92	0.96	-1.94	0.11	4.69
SEST	160.10	2.06	11.35	1.98	2.03	2.58e-08	0.68	0.01	0.99	0.99	6.13e-06	-0.57	6.94
DEST	160.30	2.26	11.56	2.32	1.63	9.81e-09	0.69	0.01	0.99	0.99	2.33e-06	-0.81	8.81
ARIMA (1,1,0)	161.00	2.97	12.27	2.11	1.98	-3.63e-09	0.71	0.01	1.00	1.00	-8.38e-07	0.15	5.21
ARIMA (1,1,1)	160.00	3.95	16.34	1.96	2.00	4.00e-08	0.71	0.01	1.00	1.00	9.21e-06	0.20	4.89
ARIMA (2,2,0)	159.00	5.30	20.42	1.63	2.27	7.26e-10	0.71	0.01	0.99	0.99	1.67e-07	-0.51	5.93
ARIMA (2,2,1)	158.00	6.42	24.48	2.14	1.98	2.23e-08	0.71	0.01	0.99	0.99	5.15e-06	-0.34	5.48
ARIMA (2,2,2)	157.00	7.56	28.54	2.06	1.98	-1.29e-08	0.70	0.01	1.00	1.00	-2.99e-06	-0.28	5.34

LR = Linear regression, MA = Moving average, MAT = Moving average with trend, SES = Simple exponential smoothing, DEST = Double exponential smoothing with trend, ARIMA = Autoregressive integrated moving average, Std RSS = Standardized residual sum of squares, MAE = Mean absolute error, MAPE = Mean absolute percentage error, MSE = Mean square error, RMSE = Root mean square error, TS = Tracking signal Sk = Skewness, Kt = Kurtosis

Both AIC and BIC favor the moving average model of order three (3-MA). The main reason of selection of this model as the winner model is parsimony. But, amazingly, Durbin-Watson test statistic as well as the ATS point towards its non-optimality. This situation demands construction and application of DLMs capable of filtering the color of noise process and employment of a simple model selection tool like ATS leading to a visible path.

1) GEWR (2, 0, 0) Type Linear Growth Model or AR (0) white Noise Process

In this study, generalized exponentially weighted regression (GEWR) type dynamic linear models of diagonal form with n = 2 are constructed and applied to get optimum forecasts (Harrison and Akram, 1983; Akram, 1987; Akram, 1992). The prior information to be supplied in case of AR(0) white noise model is as under:

$$f = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 1.61 & 0 \\ 0 & 0.32 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 5.98e{+}06 & -5.98e{+}06 \\ .98e{+}06 & 5.98e{+}06 \end{bmatrix} \quad m_0 = \begin{bmatrix} 5357.74 \\ -3.68 \end{bmatrix}$$

A discount factor $\beta = 0.535$ led to the convergence with ASL = 3.62. This value of ASL was tested. Both ATS and D-W test statistic indicate presence of coloured noise processes in AR(0) white noise model and demand restructuring of DLM by redefining f, θ, G and W values.Incorporation of a value of $\Phi 1 = 0.65$ of the autoregressive coefficient indicated by ASL = 3.62 in AR(1) colored noise process model ensures optimality.

2) GEWR (2, 1, 0) Type Model or AR (1) Colored Noise Process

The prior information for GEWR (2, 1, 0) type DLM is as under:

$$f = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad G = \begin{bmatrix} 0.3048 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 11.06 & 0 \\ 0 & 0 & 5.29 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 8626.01 & -3194.26 & -5110.80 \\ 194.26 & 2.51e{+}07 & -2.51e{+}07 \\ -5110.80 & -2.51e{+}07 & 2.51e{+}07 \end{bmatrix} \quad m_0 = \begin{bmatrix} -0.6384 \\ 2152.16 \\ 3202.58 \end{bmatrix}$$

The AR(1) model gives optimum forecast as is evident from the results in Table 2. The AIC and BIC values are lowest for the discount factor giving the above results. Hence, the models are selected accordingly but ATS also supports the same results in case of dynamic models. However, D-W test statistic does not perform satisfactorily in dynamic models.

Table 2
Dynamic Forecast Analysis for KSE 100 Index

Model	Std RSS	AIC	BIC	ASL	D-W	Bias	MAE	MAPE	MSE	RMSE	TS	Sk	Kt
AR(0)	138.77	-21.39	-12.09	3.62	0.89	-4.36e-08	0.66	0.01	0.86	0.92	-1.07e-05	0.39	3.77
AR(1)	145.26	-11.89	0.50	2.50	1.44	2.18e-08	0.67	0.01	0.90	0.95	5.26e-06	0.14	4.40

The two graphs of AR(0) white noise process and AR(1) colored noise process are as under:

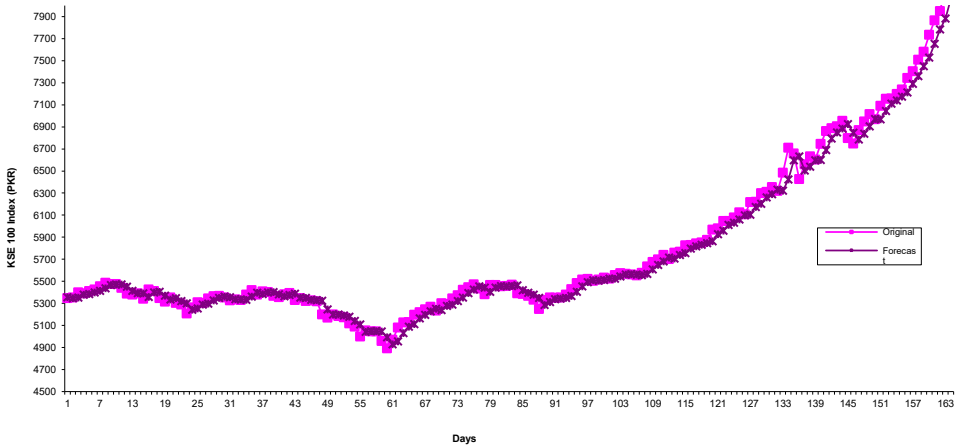

Figure 2: Graph for AR(0) White Noise Process

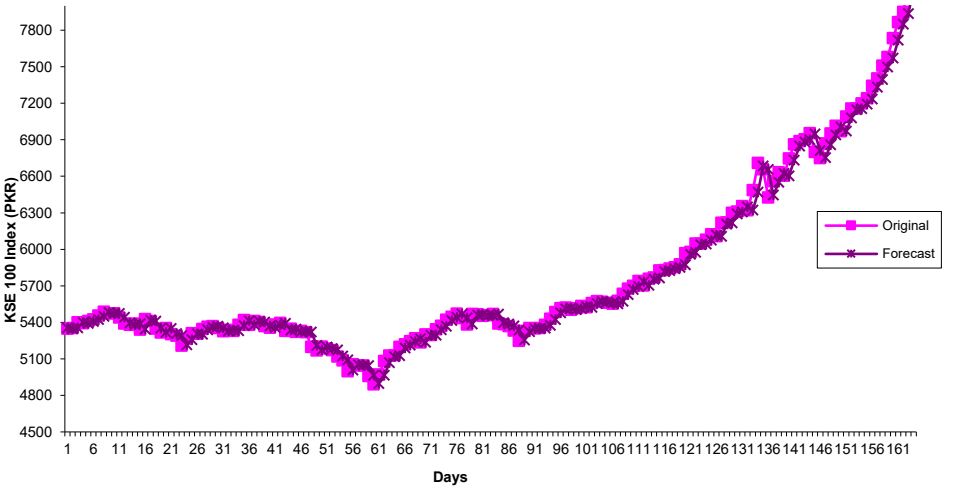

Figure 3: Graph for AR(1) Colored Noise Process

Dynamic forecasting has a definite edge over routine static forecasting. The key benefit is the definite and final identification of the winner model ensuring optimum one-step-ahead forecast after due filtration of coloured noise process, if any. ATS is an effective straight forward, parsimonious and robust tool for the identification of optimal models for time series analysis and forecasting.

The simplified DLM system presented here offers an effective solution for forecasting in noisy environments. Its transition matrix structure, Bayesian variance learning, and ATS based validation together ensure reliable and adaptable forecasting performance. The integration of ATS within a Bayesian-Kalman framework provides a robust and parsimonious method for optimal model identification. The proposed DLMs successfully filter noise and generate superior one-step-ahead forecasts, as demonstrated using KSE-100 data.

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A COMPARATIVE STUDY OF JOB-RELATED STRESS AMONG NURSES WORKING IN INTENSIVE CARE UNITS OF PUBLIC AND PRIVATE TERTIARY CARE HOSPITALS IN LAHORE

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ABSTRACT

Background: Occupational stress occurs when job demands exceed an individual's ability to cope. In hospitals, this stress is exacerbated by high workloads and complex hierarchies, particularly in intensive care units, where high nurse-to-patient ratios and constant care often lead many nurses to consider leaving the profession.

Objective: To evaluate job-related stress among Intensive Care Unit nurses working in public and private tertiary care Hospitals, Lahore.

Methodology: A cross-sectional study with nurses working in the intensive care units of public and private hospitals in Lahore. Data was collected using the HSE Management Standards Indicator Tool. A total of 214 nurses were recruited for this study.

Results: Pearson's rank correlation was utilized to analyze the relationship among the HSE management standards. The results indicated a significant relationship between all the standards of HSE management, with a p-value of less than 0.05. Additionally, there was a notable positive relationship between the working shifts and the working hours of the respondents. The graphical representation illustrated the mean values of the seven HSE management standards; lower mean values indicated reduced levels of stress, while higher mean values were associated with increased levels of stress.

Conclusion: This study adds to the growing understanding of job-related stress in the nursing profession. It concludes that the Health and Safety Executive (HSE) management standards are positively correlated. This suggests that improving the performance of one standard may lead to improvements in the others.

KEYWORDS

Work-related stress, Nurses, Tertiary care hospital, Intensive care unit.

1. INTRODUCTION

Job-related stress is defined as the response individuals may experience when confronted with job demands and pressures that exceed their knowledge and capabilities, thereby compromising their capacity to manage these challenges effectively [1].

Occupational stress has increased globally over the past three decades, negatively impacting employees' health, including nurses [2]. Health professionals are essential for enhancing access to and the quality of healthcare, with nurses and midwives making up approximately 50% of the workforce [3]. Healthcare workers are frequently subjected to high levels of stress. Medical students and healthcare workers must contend with substantial workloads and high stress levels even during their training [4]. Occupational stress has emerged as a major public health issue in recent decades, with its negative impacts on human health increasing at an alarming rate [5]. Job-related stress can be exacerbated in the hospital setting by unfavorable circumstances like high demand for work, the shifting structure, and hierarchy [6]. Stress is one of the most significant workplace issues that employees all over the world face [7]. Burnout has several negative personal and organizational consequences for healthcare professionals, including mental distress, poor health, poor working behaviors, work conflict, absenteeism, turnover, lack of job satisfaction, and medical errors, all of which can harm healthcare quality [8].

Nursing is a profession particularly susceptible to job stress, as nurses are directly responsible for patient care and managing hospital departments, in addition to various administrative tasks [9]. Intensive care units (ICUs) are a primary work setting that contributes to nurses leaving the profession worldwide, influenced by factors such as nurse-to-patient ratios, bed availability, and the demand for continuous care [10].

Occupational stress can lead dedicated younger workers to experience burnout, resulting in lower stamina and a lack of enthusiasm for their tasks. They may feel extremely fatigued, dispassionate, and disinterested in their work [11].

In nurses and other healthcare professionals, job burnout and secondary traumatic stress (STS) are the two main adverse effects of work-related stress. It has been determined that nursing is a profession with a significant amount of stress. Providing professional support and assistance to patients with diverse physical and emotional requirements, as well as to their families, puts nurses under stress [12]. Nursing is one of the top 40 most stressful jobs, with 93% of nurses experiencing daily stress, according to the US National Occupational Safety and Health Administration. A Palestinian study found that 90% of ICU nurses also face stress. Contributing factors include witnessing patient suffering, professional and administrative pressures, lack of support, neglect of emotional needs, poor management, unreasonable demands, social justice issues, and unclear duties [13]. Despite increasing evidence of the benefits of mindfulness-based interventions, research on the impact of MBSR in reducing occupational burnout and stress among nurses in medical emergency departments remains limited [14].

A study examining perceived stress among nurses was conducted in two different wards of a general hospital in Lahore, Pakistan, reporting that medical nurses experienced higher perceived stress than psychiatric nurses [15]. Keeping in view the above fact, this study aims to measure and compare job-related stress among nurses working in the ICUs of public and private tertiary care hospitals in Lahore. The identification of job-related stress would help aid in the development of appropriate interventions and the prevention of negative psychological consequences.

2. METHODS

This analytical cross-sectional study was conducted in the Intensive Care Units of public (Mayo) and private (Hameed Latif) hospitals in Lahore from July 10 to August 10, 2024. A total of 214 nurses from both hospitals were approached using a convenience sampling technique. Nurses aged 25 to 40, holding a master's degree, a Generic BSN, or a nursing diploma, with no prior psychiatric diagnoses, both married and single, and having worked in the intensive care unit for at least one year, were included in the study.

We measured job-related stress using the Work-Related Stress Indicator Tool from the UK Health and Safety Executive. This tool has 35 questions. Each question is rated on a Likert scale ranging from 1 to 5, where 1 means "never," 2 means "rarely," 3 means "sometimes," 4 means "often," and 5 means "always". Participants with stress scores above the average were classified as experiencing occupational stress. A previous study has validated this tool [16]. This 35-item Questionnaire consists of 7 subscales, control scale (items 2,10, 15, 19, 25, & 30), role scale (items 1, 4, 11, 13, and 17), demands scale (items 3, 6, 9, 12,16, 18, 20, and 22), change (items 26, 28, and 32), managers support (items 8, 23, 29, 33, and 35), peer support (items 7, 24, 27, and 31)and relationship (items 5, 14, 21, and 34) [17].

Ethical approval was granted by the Ethical Review Committee of the Public Health Department at the University of Lahore before data collection. Informed consent was obtained from all participants, and confidentiality was ensured by removing personal identifiers and securely storing the data. The collected data were entered, managed, and analyzed using SPSS version 26.0. The distribution of the data and model fit were assessed using the Student's t-test and chi-square test, respectively. Pearson's correlation coefficient was computed to define the relationship between the items in the subscales of HSE management system. Variables with a p-value less than 0.05 were considered to have a statistically significant association.

3. RESULTS

Two hundred fourteen nurses in ICUs at two hospitals in Lahore, Pakistan, were approached. The majority of nurses were females, 145 (67.7%), and married 132 (55%). The age ranges from 18 to 55, with a median age of 37 years. Half of the respondents, 106 (49.5%), were between 26 and 49 years of age. Out of 214, 38 (17.8%) nurses from Mayo hospital (public) were of diploma in nursing, and 69(32.2) were of bachelor's in nursing, while 31(14.5) nurses from Hameed Latif hospital (private) were of diploma in nursing, and 76(35.5) were of bachelor's in nursing. The majority of ICU nurses, 102 (47.7%), earned more than Rs. 41,000, while 146 (86.2%) had at least 5 years of work experience. Additionally, over one-third of the nurses, 85 (39.7%), reported working night shifts, and 106 (49.5%) worked more than 8 hours per day. Female nurses working the night shift for longer than 8 hours per week, as well as those aged 40 years or older, were significantly associated with job stress (p-value < 0.05) in both public and private hospitals (Table 1).

Table 2 displays the job-related stress scores from this study, which were measured using the HSE tool indicator. The mean scores for the factors ranged from 1.75 to 4.01, indicating that the study participants experienced moderate to high levels of stress. There was a significant association between monthly income (Rupees), working hours per day,

job schedule, and HSE tools ($p < 0.001$). Pearson's correlation coefficients were calculated between the various domains of the HSE management questionnaire (Table 3). The results indicate that all domains were significantly correlated. The highest observed correlation was between the peer support and control domains ($r = 0.691$, $p < 0.001$), while the lowest significant correlation was found between the role and demands domains ($r = -0.457$, $p < 0.001$).

Table 1
Sociodemographic Variables of ICUs in Mayo Hospital and Hameed Latif Hospital, Lahore, Pakistan, 2024 (N= 214)

Variables	Total (%)	Mayo Hospital n(%)	Hameed Latif Hospital n(%)	p-value
Gender				
Male	69(32.2)	30(14)	39(18.2)	**0.004**
Female	145(67.8)	77(36)	68(31.8)	
Age (years)				
18-25	98(45.8)	58(27.1)	40(18.7)	**0.000**
26-49	106(49.5)	41(19.2)	65(30.4)	
≥50	10(4.7)	5(2.3)	5(2.3)	
Education				
Diploma in Nursing	69(32.2)	38(17.8)	31(14.5)	0.306
BSN	145(67.8)	69(32.2)	76(35.5)	
Department				
Medical ICU	67(31.3)	37(17.3)	30(14)	0.537
Surgical ICU	72(33.6)	33(15.4)	39(18.2)	
Neonatal ICU	75(35.1)	37(17.3)	38(17.8)	
Monthly Income (Rupees)				
<40,000	47(21.9)	18(8.4)	29(13.6)	0.145
41,000-60,000	102(47.7)	52(24.3)	50(23.3)	
61,000-80,000	65(30.4)	37(17.3)	28(13.1)	
Job Experience				
< 1	39(18.2)	15(7.1)	24(11.2)	0.247
1-5	146(68.2)	78(36.4)	68(31.8)	
6-10	29 (13.6)	14(6.5)	15(7.0)	
working hours per day				
6-8	106(49.5)	49(22.9)	57 (26.6)	**0.001**
9-12	98(45.8)	51(23.8)	47(22)	
12-18	10(4.7)	7(3.3)	3(1.4)	
Job Schedule				
Morning	56(26.2)	30(14)	26(12.1)	**0.002**
Evening	73(34.1)	35(16.4)	38(17.8)	
Night	85(39.7)	42(19.6)	43(20.1)	

Table 2
Mean and Standard Deviation (SD) of Job-Related Stress Scores and their HSE Standards among Nurses, along with the Effects of Occupational Factors (n = 214)

HSE Tools and Factors	Demands	Control	Peer Support	Relationship	Manager Support	Role	Change
Gender	2.06±0.79	2.57±0.57	2.53±0.64	2.11±0.53	2.55±0.63	2.45±0.59	2.63±0.65
Age (Years)	4.00±0.90	2.96±0.85	3.63±0.88	3.22±0.31	3.63±0.88	4.01±0.92	3.65±0.44
Education	1.92±0.58	2.58±0.85	3.22±0.56	3.11±0.50	2.53±0.80	2.99±0.82	2.45±0.55
Department	3.75±0.51	3.30±0.12	3.71±0.61	3.52±0.90	3.29±0.85	3.91±0.83	3.15±0.49
Monthly income (Rupees)	2.48±0.89	2.98±0.81	2.21±0.57	1.96±0.61	2.19±0.65	2.14±0.67	2.29±0.75
Job experience	2.17±0.67	2.06±0.51	2.31±0.87	2.30±0.73	2.70±0.58	1.75±0.73	3.01±0.78
Working hours per day	2.19±0.60	2.30±0.43	2.52±0.82	3.30±0.51	2.29±0.45	2.09±0.92	2.13±0.78
Job schedule	2.39±0.59	2.77±0.91	2.07±0.48	2.10±0.61	2.02±0.49	2.63±0.89	1.95±0.51

Table 3
Correlation Matrix of Standards of HSE Management

HSE Standards	Demands	Control	Peer Support	Relationship	Manager Support	Role	Change
Demands	1						
Control	-0.402	1					
Peer Support	-0.381	0.691	1				
Relationship	0.548	-0.462	-0.312	1			
Manager support	-0.388	0.573	0.668	-0.249	1		
Role	-0.457	0.624	0.678	0.273	0.548	1	
Change	-0.301	0.59	0.579	-0.343	0.438	0.486	1

4. DISCUSSIONAND CONCLUSION

Intensive Care Units nurses from Public and Private Tertiary Care Hospitals in Lahore provided a total of 214 responses. 49.53% of participants were between the ages of 18 and 55; 49.5% were between the ages of 26 and 49. The participant's demographic information is displayed in Table 1. In total, there were 145 female nurses and 69 male nurses. From the total population, 56 worked the morning shift, 73 the evening shift, and 85 the night shift. Out of 214, 36 received a diploma in nursing, and 43 received a bachelor's degree in nursing, compared to 43 females who received a diploma and 102 who received a bachelor's degree in nursing. Table 2 presents the scores for the HSE tools along with the stress factors affecting the participating individuals. It presented the average values for seven HSE management standards; a lower average indicated a reduced level of stress, while a higher average indicated an increased level of stress. To ascertain the relationship between the HSE management standards, Pearson's rank correlation was used. There was a substantial correlation between all HSE management standards, and this test is statistically significant because the p-value was less than 0.05. The respondents' working hours and shift patterns showed a strong positive correlation.

According to recent studies, COVID-19 has raised the likelihood of occupational stress. According to studies, the mental health of the ICU personnel has been threatened by situations associated with the COVID-19 outbreak [18]. “The standards for 'control," "manager support," and 'roles' all stated that 'urgent action is needed." Control, or how much say a person has over how they conduct business, was below average. Lack of autonomy in daily tasks can dramatically lower job satisfaction for healthcare professionals. If management adopted a participatory style instead of an authoritarian one, incorporating ICU nurses in the decision-making process, this aspect would be greatly enhanced. To motivate nursing staff to pursue excellence, they should be given the power and chances to participate in decision-making processes, along with commensurate accountability [19]. It was obvious that the requirements for "demands," "peer support," and "workplace relationships" all needed to be raised. One of the main stresses and a risk factor for burnout is the demands standard, which addresses workload, work schedules, and workplace interactions [20]. According to a prior survey, 48% of nurses who left their positions did so because of poor and insufficient staffing [21]. Long hours will directly increase the likelihood of ICU nurses encountering sources of professional stress [22]. The HSE management standards have a strong positive association, which suggests that raising one standard's performance could raise the others.

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EXTENDED PARETO DISTRIBUTION BASED ON GENERALIZED ORDER STATISTICS

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ABSTRACT

The Pareto distribution is considered to be one of the most important classical models used to describe the heavy tail data. It originated from the observations of the Italian economist and sociologic sociologist Vilfredo Pareto in the nineteenth century about distribution of income and wealth. Despite its significance, its classical form often demonstrates shortcomings in its ability to capture complex tail behavior patterns observed in modern applications.

Recently, an important trend of statistical research has emerged that relies on generalized order statistics as a powerful and flexible framework for extending baseline distributions by introducing additional parameters that control tail behavior and shape flexibility. This framework represents a modern generalization of the principle underlying the transmutation process, encompassing the mathematical foundation of the quadratic transmutation map.

This paper introduces a new extension of the Pareto distribution based on the newly developed structure of generalized order statistics transmutation known as the generalized order statistics trans- muted Pareto Distribution (GOSTPD1). This model enhances the ability of the distribution to control tail heaviness and represent extreme behavior of large values by integrating the parameters that arise from the framework of generalized order statistics. This study focuses on the derivation of the most important statistical properties of the new model, including distributional forms, moments, moment generating functions, reliability, and hazard-rate functions, quantiles, and entropy. In addition, it discusses parameter estimation using maximum likelihood estimation and conducts simulations of numerical studies for different parameter values.

KEYWORDS

Generalized Order Statistics Transmutation, Pareto Distribution, Maximum Likelihood Estimation, Simulation Study.

1. INTRODUCTION

The Pareto distribution is regarded as one of the most essential heavy-tailed distributions in applied statistics. It first appeared in the work of the Italian economist Vilfredo Pareto [1] in the late nineteenth century during his analysis of the distribution of wealth and income. He observed that a small proportion of individuals held the majority of wealth, a phenomenon later known as the Pareto law. This discovery led to the formulation of the statistical model known today as the Pareto distribution. The Pareto distribution has received extensive attention due to its ability to model data characterized by extreme values, particularly in fields such as economics and income analysis [2], insurance and risk management [3], computer science and the Internet, as well as reliability studies and engineering. Heavy-tailed distributions are distinguished by their ability to assign greater weight to large values compared to light-tailed distributions, such as the normal or exponential distributions. The Pareto distribution is considered one of the most important among these distributions due to its simplicity and inherent flexibility. Nevertheless, the simple form of the Pareto distribution renders it limited in certain applications that require more flexible tail behavior or greater variability in the overall shape of the distribution. The growing interest in developing more flexible distributions has led to the emergence of several transformation mechanisms that modify the cumulative distribution function of a baseline distribution to introduce additional control parameters. Among the most prominent of these mechanisms are polynomial transformations, exponential transformations, inverse transformations, and rank-based transformations; see [4], [5]. A series of generalizations have been applied to the Pareto distribution, such as the Generalized Pareto Distribution (GPD) and various extended Pareto functions, all of which were introduced to improve its ability to capture heavy-tailed behavior [6]. The quadratic rank transmutation map introduced by [7] is considered one of the most widely used transformation mechanisms, and it has been widely used to generate transmuted distributions that exhibit a higher degree of flexibility compared to their baseline models. This idea is based on introducing one or more transmutation parameters to enhance the ability to control the skewness and tail behavior of the distribution. The quadratic rank transmutation map has been applied to the Pareto distribution in recent literature with the aim of increasing its flexibility and enhancing its capacity to model heavy-tailed data. Among the most notable applications is the model proposed by [8], which provides a general formulation of the transmuted Pareto distribution. In a subsequent step, some researchers turned to using record values as a basis for parameter inference or for predicting future observations, rather than relying on complete samples. For example, [9] demonstrated in his study how record values can be used to infer the parameters of a general distribution derived from a baseline family. With the advancement of statistical theory, generalized order statistics emerged as a comprehensive framework that unifies and extends independent models, allowing the introduction of parameters that systematically control the shape and characteristics of the distribution. In this context, the work of [10] represents a new contribution through the application of generalized order statistics to the Weibull distribution, based on this generalized ranking framework. This study demonstrated the ability of this theoretical approach to produce highly flexible models that can be further generalized to other distributions. They used the initial two generalized order statistics to derive the transmuted family of generalized order statistics as follows:

$$F_Y(x) = 1 - \frac{k+m+1}{2(m+1)}\left\{(1-\lambda)[1-F(x)]^k + \left(\lambda - \frac{k-m-1}{k+m+1}\right)[1-F(x)]^{k+m+1}\right\}, \tag{1}$$

with $\lambda \in [-1, 1]$, $k \geq 1$, and $m \in R - \{-1\}$

The present study constitutes a natural extension of this line of research, but with its application to the Pareto distribution. This paper presents an extended version of the Pareto distribution, called the Generalized Order Statistics-Based Transmuted Pareto Distribution (*GOSTPD*), to handle more complex models in the real world. In this work, we also examine the distributional functions of this new model, in addition to its statistical properties such as moments, the moment-generating function, entropy, and related characteristics. Building on this foundation, we proceed to develop point estimators for the parameters of the proposed distribution using the maximum likelihood estimation method. The cumulative distribution function (*CDF*) for the Pareto distribution is:

$$F_X(x) = \begin{cases} 1 - \left(\frac{\theta}{x}\right)^{\alpha}, \text{if } x \geq \theta \\ 0, \text{if } x < \theta \end{cases} \tag{2}$$

where $\alpha > 0$ and $\theta > 0$ are the shape and scale parameters, respectively. This cumulative form serves as the basis for the development of the transmuted and extended versions of the Pareto distribution.

2. GENERALIZED ORDER STATISTICS BASED TRANSMUTED PARETO DISTRIBUTION

This part addresses generalized order statistics derived from the transmuted Pareto distribution and its statistical characteristics, illustrating an application of the transmuted family of generalized order statistics.

2.1 Distribution and Density Functions

Theorem 1: *A random variable X is said to have a GOSTP Distribution characterized by shape and scale parameters α and $\theta > 0$ respectively, along with a transmutation parameter $|\lambda| \leq 1$, and parameters k and m that are associated with GOS, if the distribution function of X is given as follows:*

$$F_Y(x) = 1 - \frac{k+m+1}{2(m+1)}\left\{(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha k} + \left(\lambda - \frac{k-m-1}{k+m+1}\right)\left(\frac{\theta}{x}\right)^{\alpha(k+m+1)}\right\}, x \geq \theta \tag{3}$$

Proof: The CDF of Pareto distribution is given by (2) and the corresponding pdf is

$$f_X(x) = \begin{cases} \frac{\alpha\theta^{\alpha}}{x^{\alpha+1}}, \text{if } x \geq \theta \\ 0, \text{if } x < \theta \end{cases}$$

From (1) we have:

$$F_Y(x) = 1 - \frac{k+m+1}{2(m+1)}\left\{(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha k} + \left(\lambda - \frac{k-m-1}{k+m+1}\right)\left(\frac{\theta}{x}\right)^{\alpha(k+m+1)}\right\}, x \geq \theta$$

Definition 1: *The probability density function (pdf) of the GOSTP distribution can be easily written from* (3) *as follows:*

$$f_y(x) = \frac{k+m+1}{2(m+1)}\frac{\alpha\theta^{\alpha}}{x^{\alpha+1}}\left\{(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha(k-1)} + \left((k+m+1)\lambda - (k-m-1)\right)\left(\frac{\theta}{x}\right)^{\alpha(k+m)}\right\}, x \geq \theta \quad (4)$$

Figure 1 illustrates the appearance of the (*GOSTP*) distribution's *CDF* and *PDF* for various values of parameters α, θ, λ as well as different values of k and m.

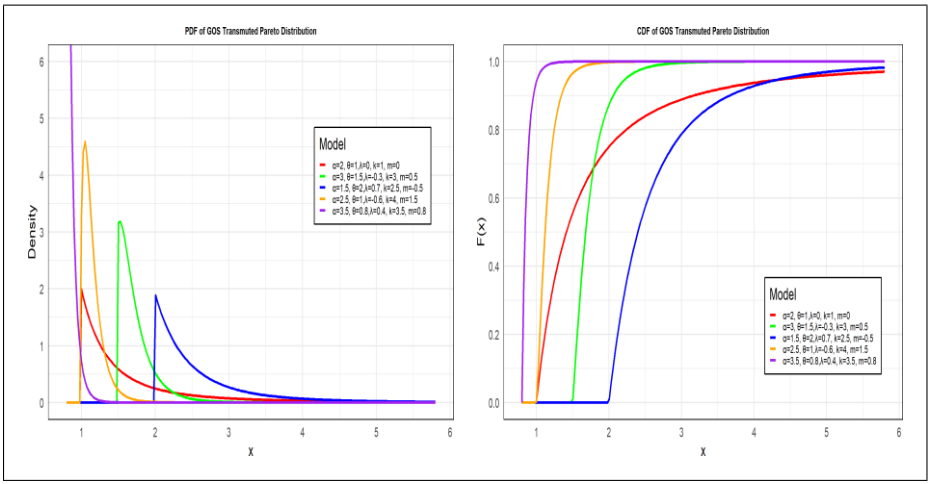

Figure 1: Behavior of the PDF and CDF for the GOST Pareto Distribution under Alternative Parameter Configurations

2.2 Statistical Properties of Generalized Order Statistics based Transmuted Pareto Distribution

In this sub-section, we have discussed some important properties Of *GOSTP* distribution. These properties are discussed below:

1. Reliability and Hazard Rate Functions

The reliability and hazard rate functions for the *GOSTP* distribution are:

$$R_Y(x) = \frac{k+m+1}{2(m+1)}\left\{(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha k} + \left(\lambda - \frac{k-m-1}{k+m+1}\right)\left(\frac{\theta}{x}\right)^{\alpha(k+m+1)}\right\}, x \geq \theta \quad (5)$$

$$h_Y(x) = h(x) \times \frac{(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha(k-1)} + \left((k+m+1)\lambda - (k-m-1)\right)\left(\frac{\theta}{x}\right)^{\alpha(k+m)}}{(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha k} + \left(\lambda - \frac{k-m-1}{k+m+1}\right)\left(\frac{\theta}{x}\right)^{\alpha(k+m+1)}} \tag{6}$$

where $h(x) = \frac{\alpha}{x}$ represents the hazard rate function of the Pareto distribution (the base distribution).

The shape of the reliability and hazard rate functions of *GOSTP* is illustrated in Figure 2, showcasing various parameter selections for α, θ, λ along with different values for k and m.

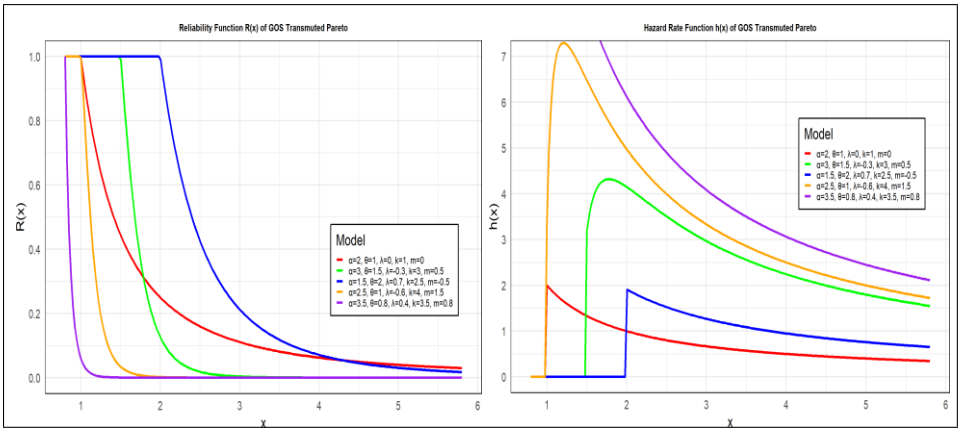

Figure 2: Plots of Reliability and Hazard Rate Functions of (*GOSTP*) for Different Choices of Parameters

2. Moments

The r^{th} moment of the distribution is obtained in the following theorem.

Theorem 2: *The r^{th} moment for the Transmuted Pareto random variable based on Generalized Order Statistics is expressed as:*

$$\mu'_r = \frac{k+m+1}{2(m+1)}\theta^r \times \left[k(1-\lambda)\left(k - \frac{r}{\alpha}\right)^{-1} + \left((k+m+1)\lambda - (k-m-1)\right)\left(k+m-\frac{r}{\alpha}-1\right)^{-1}\right] \tag{7}$$

Proof: The moments are obtained by first obtaining the probability-weighted moments, $M_{r,0,k-1}$ *and* $M_{r,0,k+m}$ of Pareto distribution as

$$M_{r,0,k-1} = \int_0^\infty \big(x(F)\big)^r [1-F(x)]^{k-1} dF(x)$$
$$= \int_0^\infty \left[\theta\big(1-F(x)\big)^{\frac{-1}{\alpha}}\right]^r [1-F(x)]^{k-1} dF(x)$$

$$= \theta^r \int_0^\infty [1-F(x)]^{k-\frac{r}{\alpha}-1} dF(x),$$
$$= \theta^r \left(k - \frac{r}{\alpha}\right)^{-1},$$

Similarly,

$$M_{r,0,k+m} = \theta^r \left(k + m - \frac{r}{\alpha} + 1\right)^{-1},$$

Using these moments in the moment equation of generalized order statistics transmuted family of distributions, see [10], the proof is complete.

Definition 2: The mean of the Generalized Order Statistics utilizing the Transmuted Pareto Distribution is expressed as:

$$\mu' = \frac{k+m+1}{2(m+1)}\theta \times \left[k(1-\lambda)\left(k - \frac{1}{\alpha}\right)^{-1} + \big((k+m+1)\lambda - (k-m-1)\big)\left(k + m - \frac{1}{\alpha} - 1\right)^{-1}\right] \tag{8}$$

The variance can also be obtained by using the expression of the r^{th} moment.

3. Moment Generating Function

The moment-generating function for the GOSTP distribution is

$$\sum_{r=1}^{\infty} \frac{t^r}{r!} \frac{k+m+1}{2(m+1)}\theta^r \times \left[k(1-\lambda)\left(k - \frac{r}{\alpha}\right)^{-1} + \big((k+m+1)\lambda - (k-m-1)\big)\left(k + m - \frac{r}{\alpha} - 1\right)^{-1}\right] \tag{9}$$

4. Quantile Function

The quantile function of *GOSTP* Distribution is obtained by solving the following equation for x.

$$\frac{k+m+1}{2(m+1)}\left\{(1-\lambda)\left(\frac{\theta}{x_p}\right)^{\alpha k} + \left(\lambda - \frac{k-m-1}{k+m+1}\right)\left(\frac{\theta}{x_p}\right)^{\alpha(k+m+1)}\right\} + p - 1 = 0$$

where, $k \geq 1, m \in R - \{-1\}, \lambda\epsilon\,[-1,1]$. This above equation can be numerically solved by using different values of α, θ, k, m, p, and λ to obtain x_p.

5. Rényi Entropy

Theorem 3: The Rényi Entropy of a random variable derived from Generalized Order Statistics based on Transmuted Pareto Distribution is expressed as:

$$I_{R(\rho)} = \frac{\rho}{1-\rho} * \log\left(\frac{(k\ +\ m\ +\ 1)}{\big(2(m+1)\big)}\right) + \rho * \frac{\log(\alpha)}{1-\rho} + \frac{\rho}{1-\rho} * \log\big(k(1-\lambda)\big) + \log(\theta) + \frac{1}{1-\rho} * \log\left(\Sigma_{j=0}^{\infty}\left[\binom{\rho}{j} * \left(\frac{\big((k+m+1)\lambda\ -\ (k-m-1)\big)}{k(1-\lambda)}\right)^{j} * \frac{1}{(\ \rho(\alpha k\ +\ 1) +\ j\alpha(m+1) -\ 1\ )}\right]\right)$$

Proof: The Rényi Entropy of a random variable with pdf $f_Y(x)$ is defined by:

$$I_{R(\rho)} = \frac{1}{1-\rho} * \log\left(\int_{\{-\infty\}}^{\{\infty\}} f_{Y(x)}^{\rho} dx\right)$$

$$I_{R(\rho)} = \frac{\rho}{1-\rho} * \log\left(\frac{k+m+1}{2(m+1)}\right) + \frac{1}{1-\rho} * \log\int_{\{-\infty\}}^{\{\infty\}} f(x)^{\rho}\big\{\ k(1-\lambda)[1-F(x)]^{k-1} + \big((k+m+1)\lambda\ -\ (k-m-1)\big)[1-F(x)]^{k+m}\big\}^{\rho} dx$$

$$I_{R(\rho)} = \frac{\rho}{1-\rho} * \log\left(\frac{k+m+1}{2(m+1)}\right) + \frac{1}{1-\rho} * \log\int_{\{\theta\}}^{\{\infty\}} \left(\alpha\frac{\theta^{\alpha}}{x^{\alpha+1}}\right)^{\rho} \left\{ k(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha(k-1)} +\ \big((k+m+1)\lambda\ -\ (k-m-1)\big)\left(\frac{\theta}{x}\right)^{\alpha(k+m)}\right\}^{\rho} dx$$

Let $t(x) = \left(\frac{\theta}{x}\right)^{\alpha}$. Then,

$$\left\{ k(1-\lambda)\left(\frac{\theta}{x}\right)^{\alpha(k-1)} +\ D\left(\frac{\theta}{x}\right)^{\alpha(k+m)}\right\}^{\rho} = \ t^{\rho(k-1)}\{\ k(1-\lambda) +\ D\ t^{m+1}\}^{\rho}$$

where, $D = (k+m+1)\lambda - (k-m-1)$

First Factor $\left(\alpha\frac{\theta^{\alpha}}{x^{\alpha+1}}\right)^{\rho}$

and the second factor $t^{\rho(k-1)}$

Hence,

$$\left(\alpha\frac{\theta^{\alpha}}{x^{\alpha+1}}\right)^{\rho} *\ t^{\rho(k-1)} = \ \alpha^{\rho}\theta^{\alpha\,\rho\,k}x^{-\rho(\alpha k+1)}$$

and

$$\{k(1-\lambda) +\ D\ t^{m+1}\}^{\rho} = \ \big(k(1-\lambda)\big)^{\rho}\{\ 1\ +\ C\ t^{m+1}\}^{\rho}$$

where, $C = \frac{D}{(k(1-\lambda))}$. Then,

$$I_{R(\rho)} = \frac{\rho}{1-\rho} * \log\left(\frac{k+m+1}{2(m+1)}\right) + \frac{1}{1-\rho} * \log\left\{ \alpha^{\rho} \theta^{\alpha \rho k} \left(k(1-\lambda)\right)^{\rho} \right\} + \frac{1}{1-\rho} * \log \int_{\{\theta\}}^{\{\infty\}} x^{-\rho(\alpha k+1)} \{1 + C\, t^{m+1}\}^{\rho} dx$$

Using Newton's generalization of the binomial theorem

$$(1 + z)^{\rho} = \sum_{j=0}^{\infty} \binom{\rho}{j} z^{j}, |z| \leq 1$$

$$z = C \left(\frac{\theta}{x}\right)^{\alpha(m+1)}$$

$$I = \frac{\rho}{1-\rho} * \log\left(\frac{k+m+1}{2(m+1)}\right) + \frac{1}{1-\rho} * \log\left(\alpha^{\rho} \theta^{\alpha \rho k} \left(k(1-\lambda)\right)^{\rho} \right) + \frac{1}{1-\rho} * \log \int_{\{\theta\}}^{\{\infty\}} x^{-\rho(\alpha k+1)} \sum_{j=0}^{\infty} \binom{\rho}{j} \left\{ C \left(\frac{\theta}{x}\right)^{\alpha(m+1)} \right\}^{j} dx$$

$$I = \frac{\rho}{1-\rho} * \log\left(\frac{k+m+1}{2(m+1)}\right) + \frac{1}{1-\rho} * \log\left\{ \alpha^{\rho} \theta^{\alpha \rho k} \left(k(1-\lambda)\right)^{\rho} \sum_{j=0}^{\infty} \binom{\rho}{j} C^{j} (\theta)^{\alpha j(m+1)} \right\} + \frac{1}{1-\rho} * \log \int_{\{\theta\}}^{\{\infty\}} x^{-\rho(\alpha k+1)} x^{-\alpha\, j\, (m+1)} dx$$

Simplify the power of x and complete integral then unified the resulting powers of θ the proof complete.

2.3 Maximum likelihood Estimation of Generalized Order Statistics based Transmuted Pareto Distribution

Consider $\boldsymbol{X1, X2, \ldots, Xn}$ to be a random sample of size *n* drawn from the *GOSTP* distribution, with $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ representing the shape and scale parameters, respectively. The transmutation parameter $\boldsymbol{\lambda}$ lies within the range $[\mathbf{-1}, \mathbf{1}]$, while $\boldsymbol{k\ and\ m}$ denote the *GOS* parameters. Let $\boldsymbol{\Theta} = (\boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\lambda}, \boldsymbol{k}, \boldsymbol{m})^{\boldsymbol{T}}$ be the $\mathbf{5 \times 1}$ vector of parameters. The overall log-likelihood function for $\boldsymbol{\Theta}$ can be expressed using (4) as

$$L(a, \theta, \lambda, k, m) = n * \ln\left(\frac{k+m+1}{2(m+1)}\right) + n * \ln(a) - \sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{n} \ln\left[k(1-\lambda) \left(\frac{\theta}{x_i}\right)^{a\, k} + \left((k+m+1)\lambda - (k-m-1)\right) \left(\frac{\theta}{x_i}\right)^{a(k+m+1)} \right]$$

We define the term:

$$T_i = k(1-\lambda)\left(\frac{\theta}{x_i}\right)^{\alpha k} + \left((k+m+1)\lambda - (k-m-1)\right)\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)}$$

The score function $U(\Theta) = (U_\lambda, U_\alpha, U_\theta, U_k, U_m)^T$ are given by:

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \frac{k^2(1-\lambda)\left(\frac{\theta}{x_i}\right)^{\alpha k} \ln\left(\frac{\theta}{x_i}\right) + (k+m+1)((k+m+1)\lambda-(k-m-1))\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)} \ln\left(\frac{\theta}{x_i}\right)}{T_i}$$

$$\frac{\partial L}{\partial \theta} = \frac{\alpha}{\theta}\sum_{i=1}^{n} \frac{k^2(1-\lambda)\left(\frac{\theta}{x_i}\right)^{\alpha k} + (k+m+1)((k+m+1)\lambda-(k-m-1))\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)}}{T_i}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \frac{-k\left(\frac{\theta}{x_i}\right)^{\alpha k} + (k+m+1)\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)}}{T_i}$$

$$\frac{\partial L}{\partial k} = \frac{n}{k+m+1} + \sum_{i=1}^{n} \frac{(1-\lambda)\left(\frac{\theta}{x_i}\right)^{\alpha k} + k(1-\lambda)\alpha \ln\left(\frac{\theta}{x_i}\right)\left(\frac{\theta}{x_i}\right)^{\alpha k} - (\lambda-1)\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)}}{T_i} + \sum_{i=1}^{n} \frac{((k+m+1)\lambda-(k-m-1))\alpha \ln\left(\frac{\theta}{x_i}\right)\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)}}{T_i}$$

$$\frac{\partial L}{\partial m} = n\left(\frac{1}{k+m+1} - \frac{1}{m+1}\right) + \sum_{i=1}^{n} \frac{(\lambda+1)\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)} + (k+m+1)\lambda-(k-m-1))\alpha \ln\left(\frac{\theta}{x_i}\right)\left(\frac{\theta}{x_i}\right)^{\alpha(k+m+1)}}{T_i}$$

The maximum likelihood estimates (MLE) $\widehat{\Theta} = (\hat{\alpha},\ \hat{\theta},\ \hat{\lambda},\ \hat{k},\ \hat{m})^T$ for $\Theta = (\alpha, \theta, \lambda, k, m)^T$ are obtained by setting the entries of the score function equal to zero and then numerically solving the resulting equations.

3. SIMULATION STUDIES

This section presents the Maximum Likelihood Estimation of parameters for previously proposed transmuted family through simulation studies.

3.1 Generalized Order Statistics based Transmuted Pareto Simulation Study

A simulation study was conducted by generating random samples of sizes $n = 50$, 75, 200, and 500 from the *GOSTP* with parameters $\alpha = 3$, $\theta = 2$, $\lambda = 0.5$, $k = 2$, and $m = 2.5$. For each sample, the parameters of the *GOSTLL* were estimated. This procedure was repeated 10,000 times to compute the average estimates and the mean square errors (MSEs). The results are summarized in Table 1. The table shows that the estimated parameter values are very close to the actual parameters. Furthermore, the estimated MSEs show a consistent decrease as the sample sizes increase. This demonstrates the effectiveness of the estimate method employed.

Table 1
MLE Estimation of the (*GOSTP*) Distribution and Corresponding Average MSEs

n	MLE					MSEs				
	α	θ	λ	k	m	α	θ	λ	k	m
50	3.8563	2.0033	0.2772	1.8961	3.0612	1.6016	0.00002	0.17695	0.6008	2.8749
75	3.6384	2.0022	0.2972	1.9012	2.8199	1.0780	0.000009	0.1599	0.4157	1.9218
200	3.5022	2.0008	0.4149	1.7496	2.7759	0.7208	0.000006	0.1114	0.2281	1.5075
500	3.1785	2.0003	0.4736	1.9520	2.6876	0.2594	0.0000002	0.0588	0.1214	0.7259

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EVALUATION OF NUTRITIONAL STATUS OF CHILDREN AGED 1 TO 5 YEARS USING WHO-2007 MID-UPPER ARM CIRCUMFERENCE CUT-OFFS: FINDINGS FROM PAKISTAN NATIONAL NUTRITION SURVEY

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ABSTRACT

This study utilizes the World Health Organization (WHO) 2007 mid-upper arm circumference (MUAC) cut-off values to evaluate the nutritional status of Pakistani children aged 1 – 5 years, particularly in resource-limited settings. This study included 55338 children (28252 boys and 27086 girls) aged 1 – 5 years old. Descriptive statistics and bivariate analysis with chi-square test and analysis of variance (ANOVA) were performed to examine the association between nutritional status with age and sex, and compare the MUAC means across different age groups. The mean (± SD) of age and MUAC were: 2.84 (± 1.29) years and 13.99 (± 2.29) cm, respectively. The highest proportion of undernourishment, 36.37% was observed in boys at 3 years of age and in girls at 5 years of age, 33.99%, while the lowest rate was identified in both genders at 4 years of age. The total proportion of undernourished children (age and gender combined) was 32.35%, with moderately and severely undernourished children accounting for 16.59% and 15.76%, respectively. The findings indicated the nutritional status results were highly significant with age group (($\chi^2 = 100.33$, $DF = 4$; p-value < 0.01) and gender status ($\chi^2 = 37.61$, $DF = 1$; p-value < 0.01).

The study concluded that the MUAC may have a simple, reliable, and rapid screening tool to assess the nutritional status of children.

KEYWORDS

Mid-upper arm circumference, Malnourished; Nutritional Status, National Nutrition Survey, Pakistani Children.

1. INTRODUCTION

Malnutrition is a serious health issue in children under the age of five. Every country in this world is affected, especially developing countries belonging to the South Asian region. In 2018, the world's most comprehensive report provided data on malnutrition in Asia. More than half of the children globally affected by acute malnutrition, approximately 26.9 million, belong to South Asia. Pakistan and India are most concerned with malnutrition in Asia: Pakistan has 10.7 million, and India's 46.6 million pediatric population is malnourished.[1] In the latest survey of Pakistan, the NNS-2018 reported that the rate of wasting has increased from 15.1% to 17.7% since 2011.[2] To evaluate the

nutritional status of children, anthropometric parameters are widely accepted and a low-cost technique. Different anthropometric variables, weight (kg), length/height (cm), and body mass index (BMI) $= \frac{kg}{m^2}$, neck circumference (NC) (cm), and head circumference (HC) in (cm) have been used in the given literature.[3,4] To diagnose acute malnutrition in children aged 6 to 60 months, this study evaluates the diagnostic accuracy of MUAC in comparison to weight-for-height Z-score (WHZ). The study, which was carried out in Rajkot, India, examines how user-friendly and straightforward it is. The study emphasizes the value of integrating MUAC with WHZ for a thorough evaluation while highlighting the potential of MUAC for detecting malnutrition, particularly in settings with limited resources.[5] To overcome this situation, researchers used MUAC as an alternative nutritional parameter, proposed as a potential proxy of weight-for-height and independence of age as well.

In community-based studies, MUAC has demonstrated superiority as a predictor of mortality in infancy compared to other anthropometric indicators. Both the advantages and disadvantages of using the MUAC as a screening measure for childhood malnutrition are highlighted in the latest study.[6] In many healthcare settings, MUAC is still underutilized despite advancements in the identification of malnutrition worldwide. The essay examines its suitability for a range of demographics, such as children in hospitals, people with long-term conditions, and telehealth environments. It also emphasizes the benefits of MUAC over conventional measures such as BMI and WHZ, but it also urges more studies to improve measuring methods and set uniform cutoff points for wider clinical use.[6] Traditionally, single cut-off values have been utilized to classify children under five years old into categories such as severe acute malnutrition (SAM), moderate acute malnutrition (MAM), and well-nourished, based on measurements of MUAC.

These cut-off values, derived from healthy children in the early 1960s, are as follows: SAM: 0 – 11.5 cm; MAM: 11.5 – 12.5 cm; normal: > 12.5 cm. Several international studies[7,8] have also described the nutritional status of children using the MUAC cut-off values. Dukhi in 2017 presented a study on the MUAC performance versus weight-for-height in South African children 0 – 59 months with acute malnutrition. This study concluded that weight-for-height is considered a more sensitive measure, and at the household level, MUAC is preferred as a quick and easy measuring tool.[9]

A similar study[10] from Bangladesh was also reported in 2022, they reported that an acute assessment of weight-for-height can often be challenging, especially in community settings. MUAC is a simple and easy-to-perform method for evaluating the nutritional status of children. In the mentioned study[10], the main objective was to evaluate the accuracy of MUAC in detecting acute malnutrition as compared to weight-for-height among children aged 6 – 59 months residing in Bangladesh. A study[11] from Hyderabad, Pakistan was to determine the frequency of low MUAC of children aged 6 – 59 months with acute malnutrition using a 135 sample size. From district Sanghar Sindh, Pakistan the authors collected cross-sectional data from 511 randomly selected children in the flood-affected areas by considering their socio-economic status.

The study[12] concluded the nutritional status of children under the age of five. The overall prevalence and severity of malnutrition in females were higher than their male counterparts. From the four developed cities in Pakistan, study[13] was carried out in 2017

the study was based on MUAC with cross-sectional data of 1474 sample size with 2 – 5 years aged children. The authors concluded that the study based on MUAC indicated that numerous children were malnourished in Pakistan. Girls were more affected than boys. A study[14] from the Children's Hospital Lahore was carried out in 2019. With a sample size of 257 children aged 6 – 59 months, they used two variables weight-for-height and MUAC to diagnose the nutritional status of children. They concluded that both indicators perform well, but in developing countries with limited resources, MUAC can be the more appropriate diagnostic method to identify malnourished children.

A study from Khyber Pakhtunkhwa (KPK) Pakistan in the severe flood-affected areas was conducted in 2021. Various associated factors of MUAC-based malnutrition have been identified in the targeted population. The results indicated that the prevalence of child malnutrition was high in flood-hit areas[15]. Another research on the validation of MUAC cut-offs of the WHO for diagnosis of the nutritional status among children under the age of five in Karachi, Pakistan was published in 2021.[16] Two more recent studies[17,18] from the national level have been published. They used the MUAC variable to identify the malnourished children in Pakistan. Both studies employ different methods i.e. find the MUAC references and another one gives the comparison of the use of different anthropometric tools in gender analysis of malnutrition.

Numerous studies in Pakistan[11–18] have evaluated the nutritional status of infants and children by using the WHO 2007[19] MUAC cut-offs. However, data on undernutrition based on MUAC using a nationally representative sample of children are often limited and should be released. As a result, we shaped this study. The primary objective of this study was to determine the nutritional status of children aged 1 to 5 years using WHO 2007 MUAC cut-offs. Furthermore, age and sex analysis associated with nutritional status has not yet been studied at the national level. Additionally, the current study gives the correlation between age and sex with their nutritional status as determined by the various methods.

2. MATERIAL AND METHODS

The data were acquired from the NNS-2018.[2] This cross-sectional study was done at the national level, and was conducted by the United Nations Children's Fund (UNICEF), with the permission of the Ministry of National Health Services, Regulations, and Coordination, Pakistan (MoNHSR&C). It was the largest collection of nutritional data covering a household-based population of children under five, adolescents, and women of reproductive age carried out in urban and rural areas of Pakistan.[2] Further details can be found in the stated Survey's report and recent studies.[4,18] The duration of the study was 10 months (April 2018 to January 2019). The sample comprised 66463 children of both sexes from birth to 5 years of age, with anthropometric variables weight (kg), length/height (cm), and MUAC (cm). Preterm infants and children with any reported chronic disease were excluded. Moreover, infants and children with any health condition affecting their growth including nutritional disorders were also excluded. The sample comprised 66463 children of both sexes from birth to 5 years of age, with MUAC (cm). There were 5709 observations of which 8.6% were missing observations: 4.4% for the boys and 4.2% for the girls. After removing the missing observations, the sample was reduced to 60754, of which 51.0% were boys and 49.0% were girls. Some studies[20,21] documented that MUAC

is not recommended among infants under six months, the reason for a lack of data on its reliability, the large variation in these age groups, and also resulting in the risk of error. Therefore, 2641 boys and 2619 girls up to six months were excluded with the recommendation of the Pakistani pediatricians in the Department of Preventive Pediatrics, Children's Hospital Multan, Pakistan. Scatter plots were used to identify the outliers in the MUAC variable. There were 156 (0.3%) outliers: 0.01% for boys and 0.2% for girls. The final study cohort consisted of 55,338 children: 28252 (51.0%) boys and 27086 (49.0%) girls.

Statistical Analysis: All analyses were conducted with the use of R version 4.4.2 (R Foundation) and SPSS (version 26.0). For descriptive analysis, the mean and standard deviation (SD) of MUAC were calculated for both sexes with each age group. To check the sex-specific mean differences in the MUAC, a two-sample t-test was used. The chi-square (χ^2) test was applied to check the significant association between two categorical variables (i.e., nutritional status with age groups with both sexes). The nutritional status of children under five years of age was evaluated by using the WHO-recommended age- and sex-specific cut-offs derived in 2007.[19] A child was considered to be

1) Undernourished: < - 2 SD
2) Moderate Under-nourished: < -3 SD to -2 SD
3) Severe under-nourished: < -3 SD

The WHO recommended age and sex-specific cut-off points are given in Table 1.

Table 1
The WHO (2007) Recommended Age and Sex-Specific Cut-Off Points for MUAC (cm)

Age (Years)[a]	Boys		Girls	
	Moderate (-2SD)	Severe (-3SD)	Moderate (-2SD)	Severe (-3SD)
1	12.5	11.6	12.1	11.1
2	13.0	12.0	12.7	11.7
3	13.5	12.5	13.3	12.2
4	13.7	12.7	13.6	12.5
5	14.0	12.9	14.0	12.8

a) age of 1-year indicates 0.5 to 1.49; 2-year: 1.50 to 2.49; 3-year: 2.50 to 3.49; 4-year: 3.50 to 4.49 and 5-year: 4.50 to 5.49

3. RESULTS

National data which represented all regions and districts of Pakistan with different socioeconomic status were utilized to evaluate the nutritional status of children. Details of WHO 2007 recommended cut-off points from 1 to 5 years are presented in (Table 1). The sample distribution of age and sex for the MUAC (cm) with mean ± SD, Table 2. Most of the children participating in this study belong to the age groups of 3 and age 4 years for both sexes, Table 2. The mean age ± SD of the total children was 2.84 ±1.29 (years), and the mean MUAC was 13.99±2.29 (cm).

The age and sex-specific mean comparison of MUAC, Table 2. Both sexes exhibited a significant increase in MUAC (cm). Boys had higher mean values than their counterparts. Significant sex differences were shown over different age groups. The one-way ANOVA results presented that there were significant age differences in the mean MUAC among both boys ($F = 533.93$; $p < 0.01$) and girls ($F = 679.64$; $p < 0.01$). The distribution of nutritional status of boys and girls was shown, respectively, Figure 1 and Figure 2. The results of the study indicated that moderately undernourished children were more prevalent than severely undernourished in both sexes. The moderate undernourished, severe undernourished, and overall prevalence were 16.6%, 15.8%, and 32.4% respectively. At age 4, boys had good health conditions compared to other age groups. Similarly, the girls had good health status in the same age groups. Age-wise nutritional status of both sexes, (Table 3, Figures 1 and 2).

Table 2
Sample Distribution of Mean MUAC of the Children of Pakistan

Age (Years)	MUAC (cm)			
	***n* (%)**	**Boys**	***n* (%)**	**Girls**
1	5678 (20.10)	13.15±2.21	5457 (20.15)	12.87±2.19
2	6174 (21.85)	13.67±2.17	5891 (21.75)	13.53±2.18
3	6711 (23.75)	14.17±2.17	6364 (23.50)	14.09±2.21
4	6539 (23.15)	14.65±2.17	6209 (22.92)	14.64±2.19
5	3150 (11.15)	14.89±2.22	3165 (11.68)	14.87±2.29
1-5	28252 (100)	14.05±2.27	27086 (100)	13.94±2.31
		F=533.930 (P- value < 0.01)		F=679.64 (P-value < 0.01)

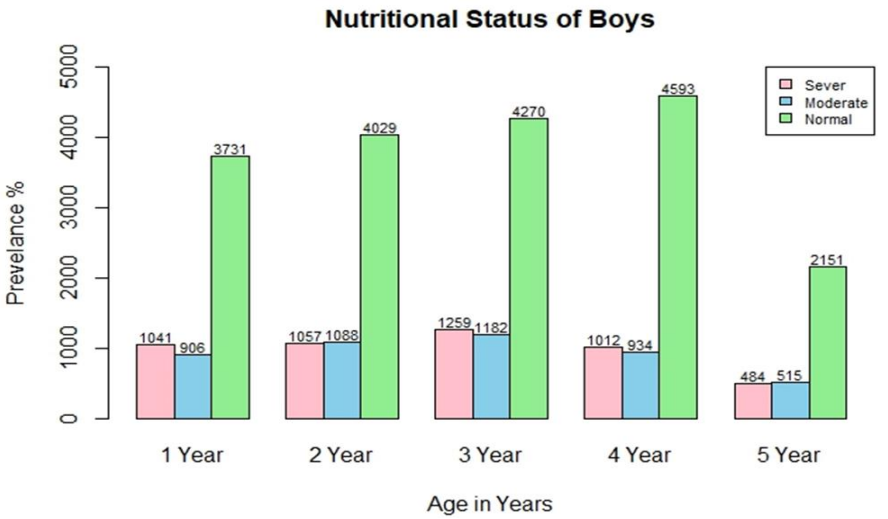

Figure 1: Distribution of Nutritional Status of the Pakistani Boys from 1 to 5 Years of Age

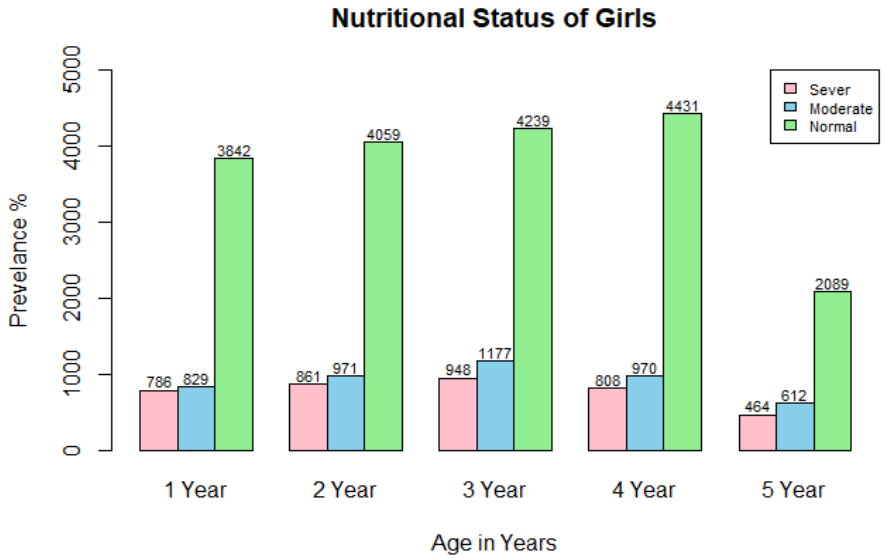

Figure 2: Distribution of Nutritional Status of the Pakistani Girls from 1 To 5 Years of Age

The Table 3 presents the statistics of undernourished Pakistani children from 1 to 5 years. The highest under-nourishment percentage 36.4% was seen at 3 years of age in boys and 34.0% at the age of 5 years in girls, and the lowest under-nourishment percentage was perceived at 4 years of age in both sexes, Table 3.

The overall percentage of undernourishment was higher in boys 17.1% than in girls 15.2%. The overall (age and gender-integrated) percentage of under-nourished infants was 32.4% of which 16.6% and 15.8% were found to be moderately and severely under-nourished, respectively. In Table 4, the overall undernutrition was observed at 32.4%, and the age-combined undernutrition among boys and girls was: 33.6% and 31.1%, respectively. The highest number of undernourished was seen at the age of 3 which was 35.0%. The relationship between age groups and sex with nutritional status was also observed by employing the Chi-square test. The results were highly significant with age-group ($\chi^2 = 100.33$, $DF = 4$; p-value < 0.01) and gender status ($\chi^2 = 37.61$, $DF = 1$; p-value < 0.01), Table 4.

Table 3
Frequency (%) of Under-Nourished based on the MUAC among Children of Pakistan by Age and Sex

Age (years)	Boys Under-Nourished				Girls Under-Nourished			
	N	Moderate	Severe	Overall	N	Moderate	Severe	Overall
1	5678	16.0%	18.3%	34.3%	5457	15.2%	14.4%	29.6%
2	6174	17.6%	17.1%	34.7%	5891	16.5%	14.6%	31.1%
3	6711	17.6%	18.8%	36.7%	6364	18.5%	14.9%	33.4%
4	6539	14.3%	15.5%	29.8%	6209	15.6%	13.0%	28.6%
5	3150	16.4%	15.4%	31.7%	3165	19.3%	14.7%	34.0%
1 – 5	28252	16.4%	17.2%	33.6%	27086	16.8%	14.3%	31.1%

Age and sex combined under-nourished:
Moderate = 17.0 %; Severe = 15.8 %; Total = 32.4%

Table 4
Nutritional status of Study Subjects based on the MUAC among Children by Age and Sex

Characteristics	Nutritional Status		
Age (Years)	Undernourished n (%)	Normal n (%)	Total n (%)
Age-Groups (Years)			
1	3562 (32.0)	7573 (68.0)	11135 (100.0)
2	3977 (33.0)	8088 (67.0)	12065 (100.0)
3	4566 (35.0)	8509 (65.0)	13075 (100.0)
4	3724 (29.2)	9024 (70.8)	12748 (100.0)
5	2075 (32.9)	4240 (67.1)	6315 (100.0)
Chi-Square = 100.3 p-value < 0.01			
Gender Status			
Boys	9478 (33.6)	18774 (66.4)	28252 (100.0)
Girls	8426 (31.1)	18660 (68.9)	27086 (100.0)
Chi-Square = 37.6 p-value < 0.01			

4. COMMENTS AND CONCLUSION

The MUAC is the most significant anthropometric measure employed throughout this study and was intended to evaluate the nutritional status of Pakistani children between the ages of 1 to 5 years. The results demonstrate the value of MUAC as an efficient method for the early identification of undernutrition in environments with limited resources and provide significant novel insights into the prevalence of malnutrition within this particular

age group. According to the findings from the recent study, 32.4% of the children were undernourished, which is the findings of NNS that Pakistan conducted in three different years.

Undernutrition rates were 38% in 2001, 31.5 % in 2011, and 28.9% in 2018, according to surveys.[2] This consistency underscores the persistence of malnutrition as a significant public health issue over the years. Several factors, such as fluctuations in sample populations, regional disparities, and the efficacy of nutrition-related interventions across time, could be responsible for the insignificant variations observed over time. Based on the recent findings, a significant number of the children in the cohort under consideration were underweight for their age group, and there was a high frequency of malnutrition generally.

Age-specific differences in the prevalence of malnutrition included greater rates of undernutrition in younger children 2 to 3 years than in older children 4 years, Table 4. This age-related tendency is consistent with the findings of research that indicates younger children are more vulnerable to nutritional deficiencies as a result of things like poor dietary intake, infectious diseases, and breastfeeding habits. In addition, the gender-specific analysis showed that girls had a greater risk of malnutrition than boys, which is in line with other research done in comparable socioeconomic environments. There are several possibilities for this disparity in sex, including varied dietary habits, cultural preferences, and differing levels of physical activity in boys and girls. [22] The results of our study are in contrast to earlier studies[6–10] carried out in South Asia, which identically revealed significant levels of malnutrition in children up to five years of age.

Global health organizations like WHO[19] and UNICEF[1] advocate the use of MUAC as a screening tool, and our study confirms the method's practicality and reliability in diagnosing malnutrition in children. In Vietnam, SAM continues to be a major cause of death for children under five. A study[23] with 4,764 children conducted place throughout 16 subdistricts scattered over four provinces in the hilly and Northern Midlands found that a cutoff of 13.5 cm was appropriate for the MUAC. When it came to identifying children whose weight-for-height z-scores were less than -3SD, this cutoff demonstrated 65% sensitivity. The results imply that WHZ and MUAC combined may improve the efficacy of therapeutic feeding programs. For children under five in these areas, the suggested MUAC cutoff of 13.5 cm, with 65% sensitivity and 72% specificity, should be used to improve and prevent SAM.

To evaluate the sensitivity and specificity of study[24] on MUAC in identifying SAM in children under five, a systematic review and meta-analysis were conducted. Eleven studies were examined by the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines using databases such as PubMed and Google Scholar. The overall sensitivity of MUAC was found to be 20.7%, ranging from 5% in Vietnam to 43.2% in India. At 97.6%, the aggregate specificity was significantly greater, and the ideal MUAC cutoff point was 13.2 cm. The study concludes that MUAC sensitivity is generally low and varies across different regions, although its degree of specificity remains high.

To investigate the validity of the WHO's global cutoff of 11.5 cm for SAM, this study [8] compared the MUAC of boys and girls aged 6 – 59 months across 22 countries in Africa, Asia, and the Americas. Principal Component Analysis was used by the researchers to

analyze data from 97,921 children who were not experiencing anthropometric inadequacy. The results showed a considerable variation in MUAC values between sexes and countries. There was no overlap between the age groups on the male and female growth curves in each cluster. According to the study, a new reference that takes population diversity into account is required to increase the precision of SAM diagnosis in children.

For instance, the Pakistan Demographic and Health Survey (PDHS) has consistently shown concerning rates of underweight, wasting, and stunting in this age group, highlighting the persistent threat that malnutrition poses to the pediatric population[25] of Pakistan. It has been observed that Pakistan has inadequate data on MUAC in children under the age of five. However, several studies[11–18] from national levels were also discussed in the past few years.

A national study[11] aims to evaluate the nutritional status of children. The objective of this study was to determine the prevalence of low MUAC in children suffering acute malnutrition from June to November 2012 at Liaquat University Hospital in Hyderabad. A total of 135 undernourished kids were evaluated; 70% of them were boys and 30% were girls. The children were 30.24 months old on average. 68.9% of the children had low MUAC, with an average MUAC of 8.85 cm. According to the study's findings, MUAC can accurately detect severe malnutrition in children.

Our study highlights, that there is a significant prevalence of malnutrition, which highlights the dire need for focused initiatives to enhance child nutrition in Pakistan. It is advised that this tool be included in regular community health evaluations due to its simplicity of use and affordability of MUAC measures. Providing healthcare professionals and carers with the appropriate training on the use of MUAC could help with the early detection and timely management of malnutrition, which could lower child mortality and improve long-term health outcomes.

Further, our results emphasize the significance of tackling the fundamental causes of malnutrition, including hunger, poverty, and limited access to healthcare. Improving the availability of healthy meals to infants, promoting exclusive breastfeeding, and increasing mother and child nutrition should be the main goals of public health initiatives. Additionally, educating the public about appropriate feeding techniques and the significance of introducing complementary foods on time could be accomplished through community-based nutrition education programs.

Strength and Limitation of the Study: This study used a cross-sectional survey that covered all the provinces and regions in Pakistan. It used a larger sample size (55338) than previous national studies[10–17] in Pakistan. Although this study offers insightful information, there are a few important limitations to be aware of. The ability of this study to prove an association between MUAC and nutritional status is limited by the cross-sectional data that we have employed. To investigate the long-term effects of early childhood nutrition on growth and development, longitudinal studies are required. Furthermore, MUAC is a good measure of evaluating the nutritional status of children, however, it is not a good measure of other types of malnutrition, such as stunting and deficits in certain micronutrients. To provide a more thorough assessment of nutritional status, future research should think about combining different anthropometric parameters.

CONCLUSION

In conclusion, this study reinforces the critical role of MUAC as a valuable tool for assessing nutritional status in children. The high prevalence of malnutrition among Pakistani children aged 1 to 5 years highlights the need for urgent public health interventions to address this ongoing challenge. By leveraging MUAC in routine health assessments and implementing targeted nutrition programs, there is potential to make significant strides in improving child health and reducing malnutrition-related morbidity and mortality in Pakistan.

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CEILING FUNCTIONS APP IN DNA GENERATION

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ABSTRACT

The DNA analysis of regularities and randomness in sequencing of amino acid in prime number form remains in big data issue for research in many generation in mathematicians from different groups and topical field in 2019 Fridman et al3 have suggest the constant f1 = 2.9200509773…… for generation of the DNA sequencing of prime number with using of recursive relation such that the celling function $h_n = P_n$. The DNA analysis will perform on SPSS version (22.0) 2 tailed test was carried out to find out the statistical difference. Our results constitute a generalization of the currently available result of fibocconi sequence Specifically DNA deals with the finite summations and is structured on PCR. A simultaneously investigate how ceiling function affect the DNA performance with SPSS software ver (22.0). The necessary measure validation studies to obtain the necessary knowledge of celling function. We introduce appropriate statistical tool. Ceiling effect causes a DNA bunching of measurable variables such that the measure becomes intensive changes in the latent variable that it is supposed to measure. The DNA twisted turn allows us to specify and justify the choice of data generators used in the simulations and the choice of the metrices used to evaluate the performance on the simulated data.

1. INTRODUCTION

Due to advancement in technology it has become easy to share information like images, data, documents, voice, videos in seconds. As this information is shared over a single band of frequency therefore it can be a questionable issue for security of personal information from the end user.

Techniques used in order to secure the information plays an important role in maintaining integrity, privacy and authentication of the data to protect from unauthorized users. In this present era of technology and digitization, security is a vital aspect; therefore, encryption is one of the ways to secure the data from being hacked. Immense spreading of communication gives rise to digitized data leading information abuse.

2. PROBLEM STATEMENT

The goal of inductive DNA bases derived on the spectral analysis of the Laplace matrix. Interestingly; these studies bears relevance to understanding of molecular evolution and

origin of life. DNA encoding is used for confusion and diffusion of image pixels. The symmetric keys are used are 32 bit ASC11key, Chebyshev chaotic key and prime key. In this research multiple research paper reviewed for creating multilevel encryption algorithm to provide several levels of security. As the chaotic system is totally unpredictable, therefore its output resemblance as noise. For initial conditions of the 3D map 3 symmetric keys in X, Y and Z dimension. One of the most important multimedia information carriers is images, containing critical and sensitive data. Securing these images from unauthorized users becomes the most important task in this digital world. Images play a vital role in many fields such as sharing information, authorization, Google maps, satellite, medical, military, etc.

3. METHODOLOGY

In this paper, multilevel encryption algorithm is developed to provide several levels of security. Among various image encryption algorithms, chaos theory has a family of techniques that are good for cryptography, as it provides high speed, reasonable computation, and good security [24]. Here, 3D chaotic logistic map is used for pixel the study has to collect quantitatively assess or monitor reconstruction DNA ceiling function. These images can be overcomes by mathematically formulation that can be easily and applied on universal truth. The quantitative measured can be classified into 2 categories consist of purely mathematically defined measures such as correlation coefficient, mean square error and so forth and the such class incorporating human visual perception of image quality into the formulation Carl Friedrich introduced the square bracket notation $[x]$ in his third proof of quadratic reciprocity (1808).

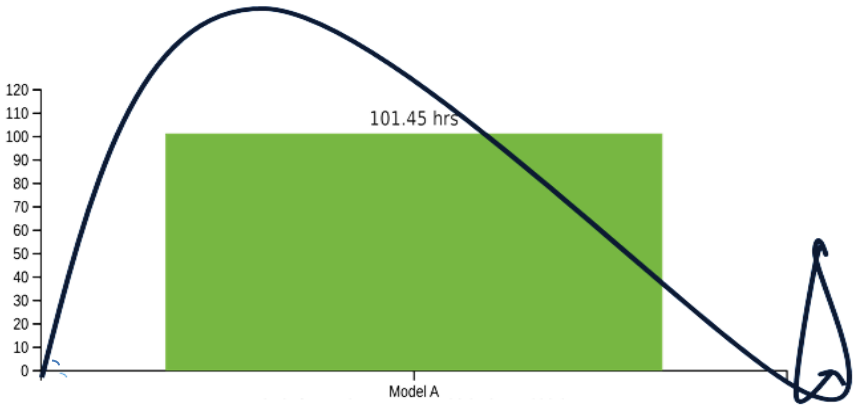


Figure 1: Graph Represent the Pixel Distribution in Normal Curve

This remained the standard[4] in mathematics until Kenneth E. Iverson introduced, in his 1962 book *A Programming Language*, the names "floor" and "ceiling" and the corresponding notations $[x]$ and $[x]$. (Iverson used square brackets for a different purpose, the Iverson bracket notation.) Both notations are now used in mathematics, although Iverson's notation will be followed in this article.

In some sources, boldface or double brackets $[\![x]\!]$ are used for floor, and reversed brackets $]\!]x[\!]$ or $]x[$ for ceiling which applicable on human DNA.

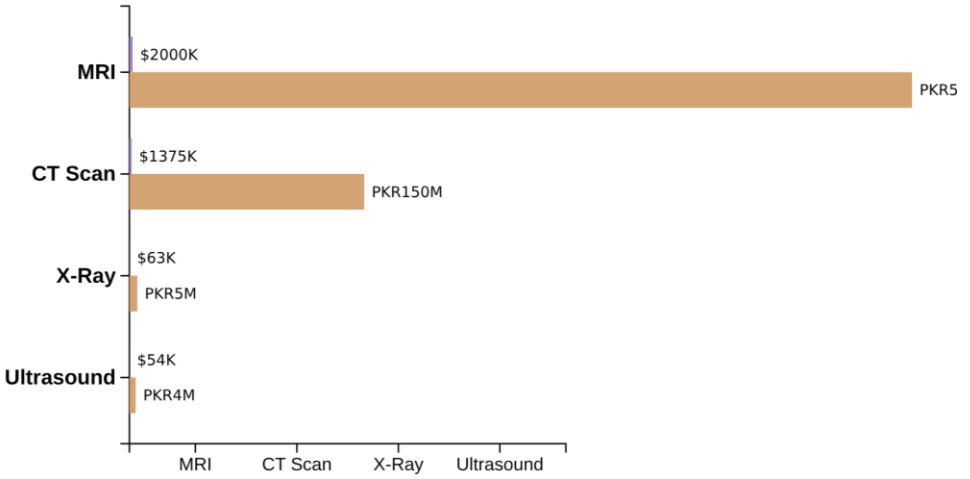
The Formula we use here for ceiling function

Let $f(\vec{x})x \in D \subseteq R^n, D = \mathcal{L}, U^{\mathcal{I}}$, ----------(1) is the feasible space.

3.1 DNA Nucleotıdes:

- Population samples by using the function (snp_vgview) presenting complete raw dataset of individual genotypes. For the population mutation parameter we take ɵ= 4 base pair complementary DNA sequence. The skeleton of a binary objects is a one pixel thick line in GA that represents the objects shape in DNA.
- Morphological transform are designed to find this structure.
- We obtained dual nucleotide for e.g. set: {AG, GA, CT, TC, AC, CA, GT, TG. AC, CA, CG, AA, CC, CG, TT} and so on which can be represents all possible mutation based on the neighboring nucleotides. Morphological transform are designed to find this structures.

The cost of medical diagnostic machines varies significantly based on technological sophistication, brand reputation, and market location. Refurbished machines often provide cost-effective alternatives.



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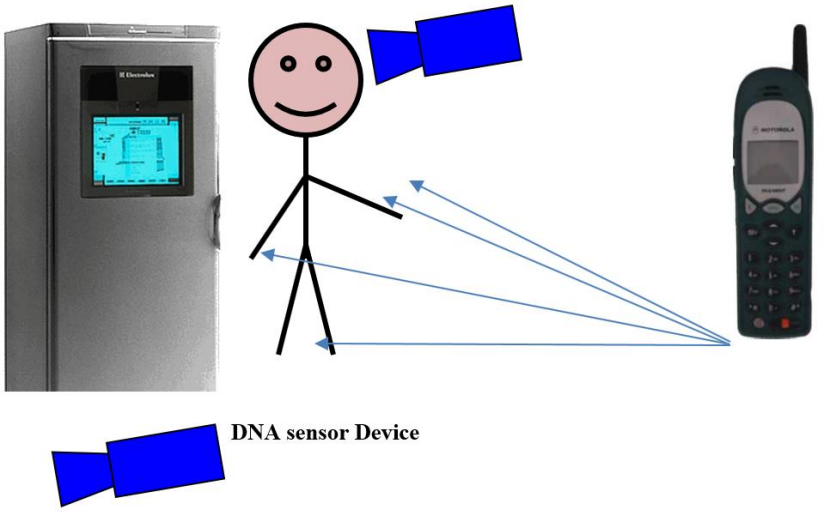

Fig: 1 X-Ray Diffraction

4. COMMENTS AND CONCLUSION

DNA ceiling function images which generated through collection of biometric coding sequences. Images allows multiple 3D graph and provides information to RNA related to any biometric machine or devices. The conventional ramp filtering into a Laplace filtering and a 2D randomly filtering which can be incorporated into routine clinical workflow and whether the information.

5. ACKNOWLEDGEMENT

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A STATISTICAL FRAMEWORK TO IDENTIFY MAJOR FACTORS CONTRIBUTING TO HEART ATTACK RISK

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ABSTRACT

This study presents an analytical investigation into the demographic and clinical factors of heart attack outcomes. Data on age, gender, heart rate, systolic and diastolic blood pressure, blood sugar, CK-MB, and troponin were considered for studying their relationship with heart attack risk. Distribution of participants in different age and gender groups was considered for comparisons, presenting higher representation of males and concentration in middle-aged and older adults. Chi-square testing documented a significant association of blood pressure with age and gender, while other clinical factors were without meaningful variation across demographic groups. Comparisons across multiple bar charts indicated a clear trend in heart rate, blood pressure, blood sugar, CK-MB, and troponin in subjects presenting across age and gender groups. Logistic regression analysis identified age, gender, heart rate, and troponin as positive predictors for heart attack likelihood, while systolic pressure, diastolic pressure, blood sugar, and CK-MB were not found to be significant variables. The findings show that age, gender, and important biomarkers play a major role in heart-attack risk. This means more detailed research is needed to improve early detection and prevention of heart disease.

KEYWORDS

Heart Attack; Cardiovascular Risk Factors; Systolic Blood Pressure; Diastolic Blood Pressure; Blood Sugar Level; Heart Rate; Logistic Regression

1. INTRODUCTION

"Heart attack, or myocardial infarction, is a life-threatening condition that occurs when the supply of oxygenated blood to a portion of the heart muscle is suddenly cut off," thereby causing the "affected portion of heart muscle to become damaged or die." "The applications of understanding heart attacks are to emergency care, medication, recovery, and prevention."

According to an article by Mira in (2025), there are many different types of symptoms associated with experiencing a heart attack, including: Chest Pain Shortness of Breathm Sweating Profusely Other Serious Symptoms All these symptoms need immediate medical attention. One can help to prevent future occurrences of heart attacks by changing their

lifestyle choices to include a balanced diet, engaging in physical activities that encourage cardiovascular fitness, and working to reduce any unnecessary levels of stress. In addition, research has demonstrated that understanding the causes of heart attack risk can help to develop prevention plans that are based on sound research methods such as binomial logistic regression.

According to Zulkiflee & Rusiman (2021), the human body consisted of various body organs that had vital functions. The most critical organ that functioned to pump blood into our lungs was the heart. The epidermis was overlaid with a tissue membrane known as the pericardium, and the rib cage covered the heart. Oxygenated and deoxygenated blood were kept apart in its four chambers. Plaque accumulation in the blood vessels and arteries led to heart disease. The waxy substance known as plaque was created by minerals, lipid molecules, and cholesterol. Rose et al. (2023) noted that heart disease has posed various significant health challenges, especially in terms of its precise detection and identification in individuals. Despite the availability of several medical devices for predicting heart disease, the authors contended that these tools were frequently very costly and did not provide adequate efficiency to reliably assess the likelihood of developing cardiac issues.

Cardiovascular risk factors refer to traits or behaviors that elevate the risk of developing cardiovascular disease (CVD). These factors are divided into non-modifiable (such as age and family history) and modifiable (including smoking, high blood pressure, lack of physical activity, and poor diet). Understanding these factors is crucial for identifying individuals at elevated risk and for implementing lifestyle modifications and medical interventions aimed at decreasing the chances of future incidents like heart attacks or strokes.

Rahaman et al. (2025) showed that CVDs are a group of disorders affecting the heart and blood vessels, which includes diseases such as coronary artery disease, myocardial infarction (heart attack), stroke, arrhythmias, and congestive heart failure. Mira (2025) reported that cardiovascular diseases (CVDs) were one of the leading causes of death globally, killing millions each year. Atherosclerosis was the main cause of these diseases, this is a disease affecting the arteries characterized by stiffening and inflammation due to the invasion by high levels of cholesterol and lipids from culturally processed food. This mechanism greatly increased the risk of heart attacks and other major cardiac events.

Key risk factors encompassed age, gender, hypertension, diabetes, obesity, smoking, and lack of physical activity. Kartiosuo et al. (2024) stated that cardiovascular diseases (CVD) generally has an early life origin, as the atherosclerotic process can initiate in childhood. High BMI, high blood pressure, abnormal cholesterol, raised triglycerides, and smoking during childhood are powerful predictors of adult cardiovascular disease. These risk factors tend to track from childhood into adulthood, where children with high levels usually continue into adulthood with the same risks.

Systolic blood pressure, such as 120/80 mmHg, is the peak pressure in your arteries during a heartbeat. It is used to detect and track hypertension, or elevated blood pressure, which may be a sign of a higher risk of heart disease, stroke, and other illnesses. It is also used to direct treatment for disorders ranging from shock to heart disease. Systolic blood pressure is used to **diagnose high blood pressure, identify cardiovascular disease risk,** and **monitor heart and blood vessel health**.

Systolic blood pressure (SBP) represents the highest pressure in the arteries during a heartbeat, according to Lam et al. (2021). While SBP beyond 160 mmHg raises the risk of cardiac injury, intraoperative SBP below 100 mmHg might harm organs, particularly in patients with pre-existing hypertension. For proper organ perfusion and improved surgical results, SBP must be kept within an ideal range. Gao et al. (2024) found that the estimated annual percentage change of mortality of stroke attributable to high systolic blood pressure varied by stroke subtypes, with a slight increase for ischemic stroke (IS), a modest decrease for intracerebral hemorrhage, and a notable decrease for subarachnoid hemorrhage (SAH). Mortality due to high systolic blood pressure from IS in males has increased significantly for cohort and period rate ratios. Nozato et al. (2025) stated that with ageing, that systolic blood pressure (SBP) rises progressively for arterial stiffening and vascular regulatory mechanisms are altered. A target SBP <130mmsHg is recommended for those <75 years and <140 mmHg for 75+ years. There is strong evidence that reducing SBP <130mmHg in adults aged ≥ 75 leads to a significant reduction in the number of cardiovascular events, mortality rate for ALL causes and cardiovascular mortality rate without increasing the incidence of serious adverse events. Therefore, the findings highlight the value of intensive and individualized blood pressure management in older hypertensive patients. Diastolic blood pressure measures the lower value in a blood pressure test, which indicates pressure in the arteries when the heart pauses between beats. It indicates minimum pressure in the arteries before the next heartbeat, when the heart's filling chambers are filled with blood. The use of diastolic blood pressure in testing applies in diagnosing high blood pressure and estimating risks involved in heart disease, since it indicates pressure in the arteries when the heart pauses between beats. Matsuzawa (2022) pointed out that “Diastolic Blood Pressure (DBP) was an indication of the pressure exerted upon the arteries during heart relaxation, a high DBP indicated an elevated systemic vascular resistance.” Initially in hypertension, there was an increase in DBP, which was significantly associated with organ damage, especially during the compliant phase of the arteries. Lepoittevin et al. (2025) also stated that ”the lowering of the threshold of diastolic BP from 85mmHg to 70mmHg is a major change in concept, but this change would seem to lack the strength of evidence found in the changes in systolic BP.” It has been found in observational studies that there is uncertainty in establishing an elevated risk of CVD in individuals with a diastolic BP < 80mmHg, with a U shape being indicated in some analyses. Excessively low diastolic blood pressure (DBP) is concerning for older adults and individuals with existing atherosclerosis because it may restrict coronary perfusion and thus increase one's susceptibility to cardiovascular diseases according to Beddhu et al. (2018) investigated the effect of DBP and showed its significant benefits in reducing cardiovascular risk. Although there was a strong agreement on the treatment of high DBP, the practice of bringing down DBP to very low levels remained a matter of debate. Previous studies also showed a J-shaped relationship showing that myocardial infarction risk was lowest with DBP levels from 85–90 mmHg, below and above which the risk started to rise. All these findings gave cause for concern that a low baseline DBP would reduce the protective effects of an aggressive systolic BP reduction.

The level of glucose or sugar present in your blood determines your blood sugar level. Testing for your blood sugar level can be used to screen for and diagnose conditions such as prediabetes and diabetes, and to help you manage the condition through lifestyle changes

and treatment. Sriwahyuni et al. (2021) reported that diabetes mellitus was characterized by high blood glucose levels resulting from defects in insulin secretion, insulin resistance, or both. Optimal control of glycemia relied on regular blood glucose tests coupled with therapeutic measures. They also highlighted that physical activity, especially aerobic exercise like brisk walking, contributed significantly to improving insulin sensitivity and maintaining blood glucose levels in a non-fluctuating state. Evidence also suggested that routine activities involving as little as 30 minutes of walking or 20 minutes of brisk walking contributed meaningfully to glucose regulation in people with diabetes. Bonna et al. (2023) described blood sugar using FBS and 2-hour post-glucose blood sugar, pointing out the fact that in most instances, increased BMI was accompanied by raised blood sugar. It pointed out the high risk of excessive blood glucose and insulin resistance associated with excess weight.

Heart rate is the number of heartbeats within a minute-it is also commonly referred to as pulse. It is an indicator of the workload of the heart and how efficiently it circulates blood throughout the body. Your emotions, exercise, and state of health can be factors in altering your heart rate. Heart rate applications are really endless, but are primarily health and fitness tracking-monitoring daily activities, managing stress, assessing sleep quality, and optimizing training intensity. Duijvenboden et al. (2025) concluded that impaired cardiac autonomic function, including impaired heart rate regulation, was common in PD and may often precede the motor symptoms. Two important markers of this are heart rate recovery (HRR), reflecting parasympathetic activity, and heart rate exercise (HRE), and reflecting sympathetic activation, both easily measured and clinically informative and may be early indicators of Parkinson's disease (PD). Mejía et al. (2020) indicated that heart rate variability (HRV) serves as an indicator of autonomic nervous system function and is associated with cardiovascular issues such as hypertension, diabetes, and heart failure. Pulse rate variability (PRV), which is assessed through photoplethy smography (PPG), offers a straightforward, non-invasive method for ongoing monitoring. While PRV yields information akin to HRV, the connection between the two is not completely understood in non-stationary or unhealthy states. PRV is being utilized more frequently in the evaluation of cardiovascular health.

Logistic regression is a statistical technique employed for binary classification tasks, predicting the likelihood of a discrete outcome (such as 0 or 1, yes or no) based on one or more independent variables. That includes finance-credit scoring and risk assessment, disease diagnosis and prediction of treatment response in healthcare, customer churn prediction and marketing in business, and jobs such as text classification and spam detection in natural language processing. Dey et al. (2025) emphasized that logistic regression (LR), though developed in the nineteenth century, and is widely used today for categorical outcomes. It is especially applied in binary classification to estimate the probability of an event based on one or more predictors, helping analyze relationships between independent factors and dichotomous outcomes. Elkahwagy et al. (2024) showed that logistic regression was used in biomarker research to predict categorical outcomes, such as disease presence, based on one or more factors. It allows calculation of key diagnostic measures, including sensitivity, specificity, Positive Productive Value (PPV) and Negative Productive Value (NPV) and helps assess the predictive ability of biomarkers for clinical application. Logistic regression is a statistical technique that predicts a discrete

(categorical) result based on one or more independent variables, according to Zulkiflee & Rusiman (2021). The result variable may be mixed, dichotomous, permanent, or discrete. To investigate trends of antidepressant use at a tertiary care facility, for example, researchers used binary logistic regression.

2. STUDY OBJECTIVES

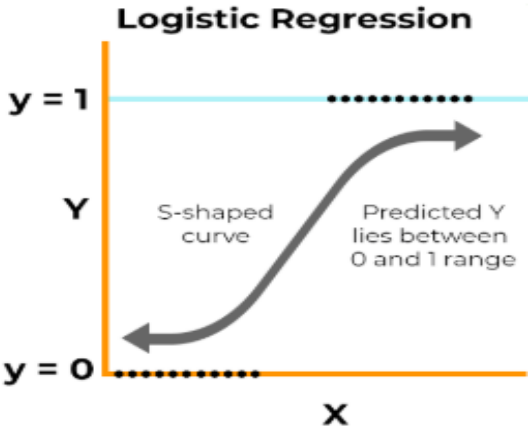
This study aims to achieve the following objectives:

1. Recognize risk factors: Understand the demographic and health elements that influence the likelihood of heart attacks, including age, gender, heart rate, systolic blood pressure, diastolic blood pressure, blood sugar, ck-mb, and troponin.
2. Employ statistical methods such as binary logistic regression, chi-square tests, and multiple bar-chart comparisons to investigate the relationship between each variable and heart attack outcomes, considering both age wise and gender wise comparisons.
3. Determine the degree to which each independent factor predicts the risk of a heart attack using the binary logistic regression model, which yields precise odds and probability estimates.

3. METHODOLOGY

Logistic Regression:

Logistic regression is a statistical technique employed for predictive analysis, which estimates the likelihood of an event happening based on one or more independent variables. This method is applicable when the outcome variable is categorical, indicating it has a finite number of outcomes, typically two (binary), such as "yes" or "no". The model applies an S-shaped curve to forecast the probability of a particular outcome, which will invariably fall between 0 and 1.



Logistic Regression Model:

Logistic regression is a **statistical model** used to predict a **binary outcome** (Yes/No, 0/1, Success/Failure). If the dependent variable is Y (which can be 0 or 1) and independent variables are $X1, X2 \ldots, Xn$ the logistic regression model is:

$$Logit(p) = ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n$$

where:

- $p = P(Y = 1 \mid X)$ → the probability that the dependent variable is 1.
- $Ln\left(\frac{p}{1-p}\right)$ → log-odds or logit function.
- $\beta_0 + \beta_1 + \beta_2 + \ldots\ldots\ldots + \beta_n$ → coefficients estimated by the model.

Logistic Regression Function:

The logistic function converts the linear combination of variables into a **probability between 0 and 1:**

$$p = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots\ldots\ldots + \beta_n X_n)}}$$

This gives a probability between 0 and 1.

- If $p > 0.5$ → predict Y=1Y = 1Y=1
- If $p \leq 0.5p$ → predict Y= 0

Chi-Square Distribution:

A chi-squared test (also known as chi-square or χ^2 test) is a statistical hypothesis test used to determine whether there is a significant association between two categorical variables. It is primarily used in the analysis of contingency tables when the sample sizes are large. The formula for the chi-square test is:

$$\boldsymbol{\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}}$$

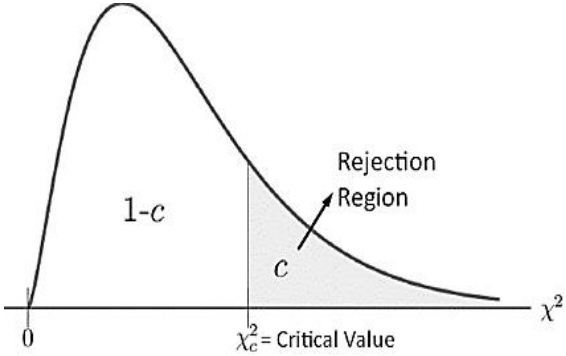
where:

$\boldsymbol{\chi^2}$: The chi-square test statistic

$\sum$: The summation symbol, which means you add up the values for all categories

$\boldsymbol{O_i}$: The observed frequency (the actual count in each category)

The expected frequency (the count you would expect in each category)



Contingency Table of Chi-Square Test:

	Archery	Boxing	Cycling	
Female	35	15	50	100
Male	10	30	60	100
	45	45	110	200

Multiple Bar Chart:

The Multiple Bar Diagram is used to compare two or more variables such as revenue and expenditure, import and export for different years, marks obtained in different subjects in different grades, and so on. It is often referred to as a **Compound Bar Diagram.** The method for creating multiple bar diagrams is the same as for making a Simple Bar Diagram. However, to distinguish the bars from each other, different bars are differentiated by different shades or colors.

4. RESULTS AND DİSCUSSİON

Table 1
A Chi-Square Contingency Table was used to Check the Effect of Age along with other Factors

Sr. No.	Attributes	DF	Chi-Square	P-Value	Remarks
1	Age*Gender	4	6.000	0.199	Insignificant
2	Age*Heart rate	36	42.000	0.227	Insignificant
3	Age*Systolic BP	12	14.000	0.0301*	significant
4	Age*Diastolic BP	36	46.000	0.0227*	significant
5	Age*Blood Sugar	30	35.000	0.243	Insignificant
6	Age*CK-MB	42	48.000	0.243	Insignificant
7	Age*Troponin	30	35.000	0.243	Insignificant
8	Age*Result	36	42.000	0.227	Insignificant

Table 1, show that the Chi-square analysis evaluated eight associations based on age. There was no significant correlation between age and gender ($p = 0.199$), heart rate ($p = 0.227$), blood sugar ($p = 0.243$), CK-MB ($p = 0.243$), troponin ($p = 0.243$), or the final diagnostic outcome ($p = 0.227$). This implies that these factors appear not to change much between age groups. In contrast, two factors systolic blood pressure ($p = 0.0301$) and diastolic blood pressure ($p = 0.0227$) had a significant association with age, implying that the trend in blood pressures significantly alters with age. In summary, only the blood pressure metrics revealed age-related differences, while all other clinical factors showed no significant variation across different age categories.

Table 2
A Chi-square contingency table was used to Check the Effect of Gender along with other Factors

Sr. No.	Attributes	DF	Chi-Square	P-Value	Remarks
1	Gender*Heart rate	16	20.000	0.220	Insignificant
2	Gender*Systolic BP	20	24	0.0242*	significant
3	Gender*Diastolic BP	9	11.25	0.0259*	significant
4	Gender*Blood Sugar	12	15.000	0.241	Insignificant
5	Gender*CK-MB	16	20.000	0.220	Insignificant
6	Gender*Troponin	12	15.000	0.241	Insignificant

Table 2, show that the Chi-square analysis that tests the association of gender with different clinical attributes. The analysis yields significant relations with respect to gender for the variables systolic blood pressure ($p = 0.0242$) and diastolic blood pressure ($p = 0.0259$). This means that there is a significant difference in the distribution of blood pressure in males and females. On the other hand, heart rate ($p = 0.220$), blood sugar ($p = 0.241$), CK-MB ($p = 0.220$), and troponin levels ($p = 0.241$) are not significantly related to gender. Hence, these can be said to be relatively invariant for both genders. Overall, the results infer that only the measures of blood pressure differ by gender, while all other measures of clinical indicators do not exhibit significant differences by gender.

Figure 1(a) illustrates that the incidence of heart attacks differs among various age groups, with the most significant occurrences found in the 46–60 and 61–75 age ranges. This might indicate that middle-aged and older adults are more at risk. The cases among those aged between 31–45 years were moderate, and the ≤30 and >75-year age brackets showed very few cases, indicating a lower risk at these extremes. With regard to gender distribution, males in every category outnumber females, which could indicate that men are most susceptible to heart-related conditions throughout the population under study. Figure 1(b) shows the variations in heart rate per minute as per various age groups. The 50–59 and 60–69 age groups have the highest heart-rate frequencies, which means middle-aged to older subjects tend to show a high heart rate. The 40–49 and 70–79-year-old groups stand midway, while less than 30, 30–39, and greater than 80-year-old groups fall in lower frequencies, inferring at the extremes of these ages, there is lower heart-rate activity. For the gender distribution, most of the age groups have more males than females; this would suppose that men may have higher heart-rate readings among the population studied. Figure 1(c) shows that the systolic blood pressure is represented differently in the various age groups for both males and females. Age categories 50–59 and 60–69 years exhibited

the highest levels of systolic pressure, indicating that this middle to older age group has the highest propensity for blood pressure increases. The 40–49 and 70–79-year-old groups exhibited moderately high values, showing a gradual increase followed by a moderate decline in the older decades of life. The age categories <30, 30–39, and >80 years had the lowest systolic pressure, indicating that the very young adult and those in the upper extremes of age exhibit relatively low systolic pressure activity across the dataset. Lastly, male subjects feature more than female subjects in the majority of age groups, suggesting that men in this sample have relatively higher systolic pressure. Figure 1(d) indicates that the diastolic blood pressure varies by age group. The 60–69 age bracket contains the highest frequency of diastolic values, followed by high diastolic values in the 50–59 and 70–79 age brackets, reflecting higher values in middle to older adults. The 40–49 age bracket is in the middle, while 18–29, 30–39, and >80 are characterized by a lower frequency of diastolic values, reflecting relatively lower values during younger and extremely older ages. Males appear marginally more frequent than females across most age brackets, which implies that, within this dataset, men may have comparably raised diastolic values. Figure 1(e) represents the variation in blood sugar level across different age groups. The highest values are for the 60–69 age group, while 50–59 and 70–79 years also reveal very high values, which means that middle to older adults have higher blood sugar levels. The variation in the 40–49 age group falls in the middle. However, the 18–29, 30–39, and >80 age groups have lower frequencies of blood sugar, reflecting relatively lower values in younger age and very old individuals. Across most age ranges, **males appear more frequently than females**, hinting that men may exhibit slightly higher blood sugar levels in this dataset. Figure 1(f) the graph shows that CK-MB levels change with age. Younger people (below 40) have the lowest levels. The levels rise in older adults and are highest in the 51–60 and 71–80 age groups. After age 90, the levels drop again. The graph also has more data for men than women, which may suggest that men have higher CK-MB levels. Overall, CK-MB is highest in middle and older age, but it decreases in very elderly people. Figure 1(g) the graph shows that Troponin levels change with age. Younger people (below 40) have the lowest levels. The levels rise in older adults and are highest in the 50–60 and 70–80 age groups. After age 80, the levels drop again. The graph also has more data for female than men, which may suggest that female have higher Troponin levels. Overall, Troponin is highest in middle and older age, but it decreases in very elderly people. Figure1 (h) the graph illustrates that test outcomes vary significantly with age. In the 0–29 age group, both negative and Positive results are quite low, suggesting a minimal likelihood of the condition. From ages 30 to 49, both results increase, with Positive results rising more sharply, indicating a moderate increase in risk. The 50–59 and particularly the 60–69 age groups exhibit the highest Positive results, indicating that individuals in these age ranges have the greatest likelihood of testing positive. In the 70–79 age group, Positive results remain elevated but are slightly lower than the peak observed earlier. For those aged 80 and above, both results decline once more, implying a moderate chance of the condition in later years.

Heart rate values clearly differ between males and females across age groups, as seen in Figure 2 (a). Males consistently exhibit higher heart rate readings than females across all age groups. Both genders' heart rates are lowest in the 50–59 age range, although they gradually increase in older adults. The 60–69 and 90+ groups are where the male heart rate peaks, while the 60–69 group is where the female heart rate peaks. According to the graph,

heart rate normally rises with age before leveling or slightly declining in later years, and it tends to be higher in men than in women at all ages. Figure 2 (b) the graph shows that males have higher systolic blood pressure levels than females in all groups. The highest values for both genders appear in the **140–159** range, where men peak strongly. Males also dominate the **100–119** and **160–179** groups. The lowest values for both genders are in the **<100** and **>180** ranges. Overall, high systolic blood pressure is more common in men, especially in the middle range (140–159). Figure 2 (c) the graph shows that males have higher diastolic blood pressure levels than females in all groups. The highest values for both genders appear in the **80-89** range, where men peak strongly. Males also dominate the **60-69** and **70-79** groups. The lowest values for both genders are in the **<60** and **≥90** ranges. Overall, high systolic blood pressure is more common in men, especially in the middle range (80-89). Figure 2 (d) the graph shows that males have higher blood sugar levels than females in all groups. The highest values for both genders appear in the **<100** range, where men peak strongly. Males also dominate the **100-125** and **126-199** groups. The lowest values for both genders are in the ≥ 200 ranges. Overall, high blood sugar level is more common in men, especially in the middle range (<100). Figure 2 (e) the graph indicates that males have greater CK-MB levels than females. Males exhibit a pronounced peak in the 0–5 range, where the highest values are found. The 5–20 group is likewise dominated by men. Both genders' values decline in the upper CK-MB ranges (20–50, 50–100, 100–300), while males' levels stay higher. In general, males are more likely to have increased CK-MB levels, particularly in the 0–5 and 5–20 ranges. Figure 2 (f) the graph shows males have higher troponin levels than females in all groups: 0–0.01, 0.01–0.05, 0.05–0.50, 0.50–1.0, and >1.0, with the highest peak in the 0–0.01 group. Males consistently exceed females across all ranges.

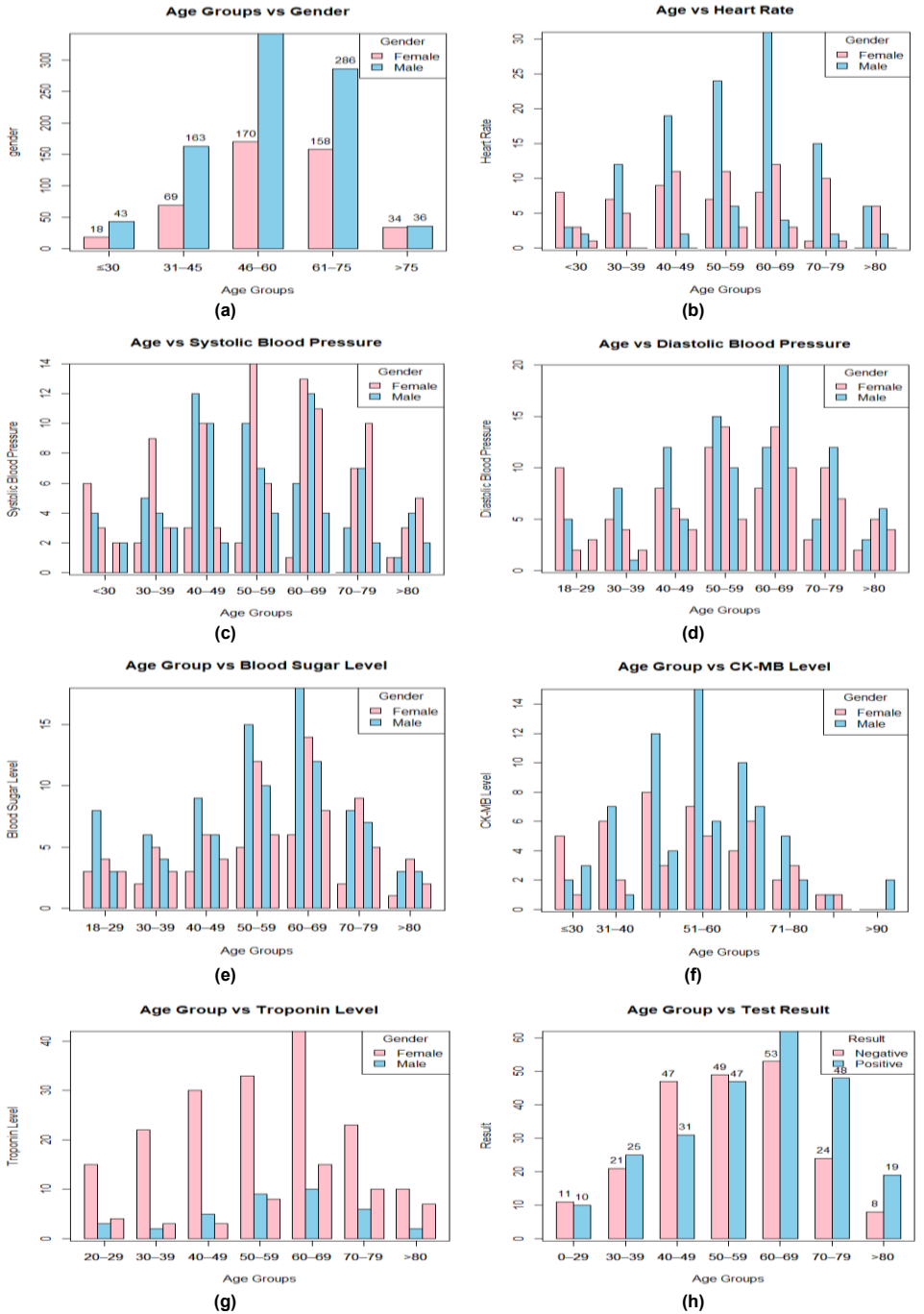


Figure 1: Heart Attack Factors Across Age Groups

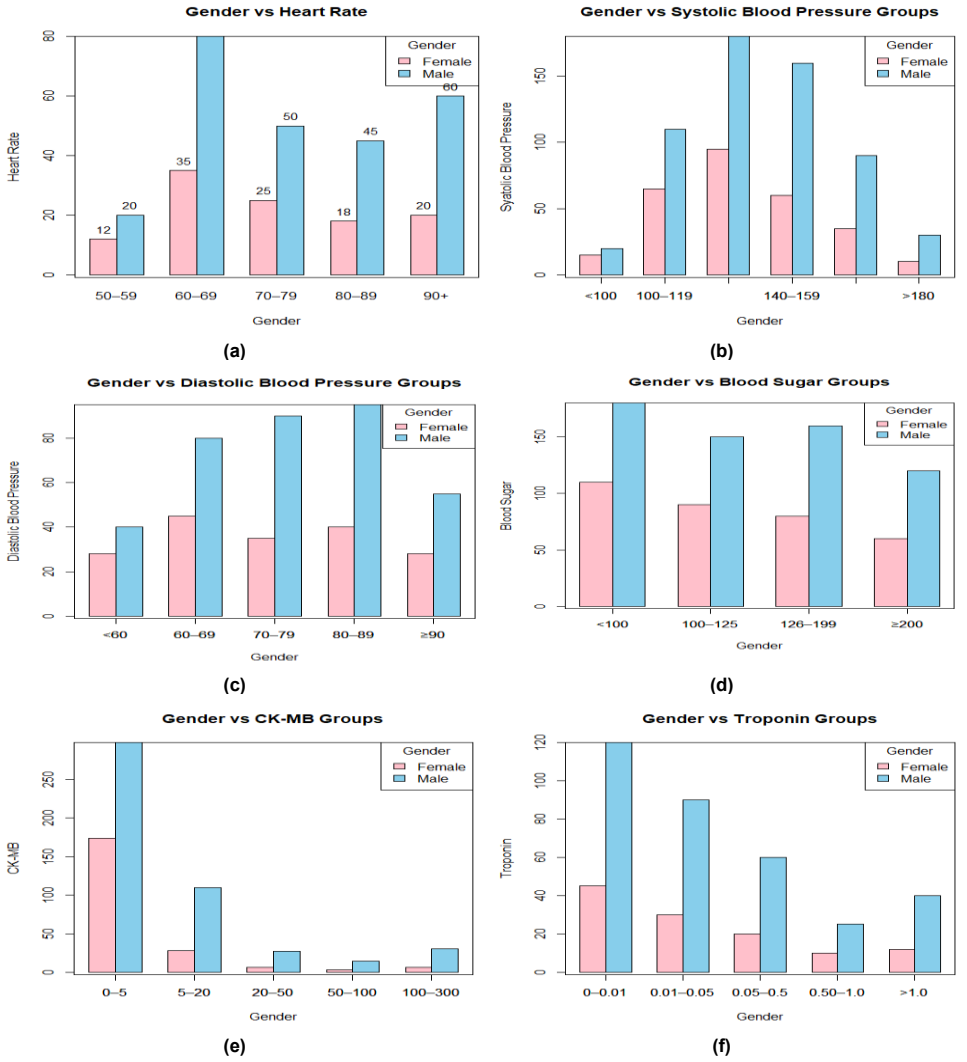


Figure 2: Heart Attack Factors Across Gender

Omnibus Tests of Model Coefficients for Blood Clot Risk Analysis

Step	Chi-Square	Degree of Freedom(df)	P-Value (sig.)
Step 1	648.982	8	.0001
Block	648.982	8	.0001
Model	648.982	8	.0001

The Chi-square statistic of 648.982 (considering all steps: Step, Block, and Model) with 8 degrees of freedom suggests that the overall model is highly statistically significant. Additionally, the p-value (< 0.001) for all tests is significantly lower than the conventional significance threshold of 0.05, indicating that the model, along with each step and block, plays a crucial role in predicting the occurrence of blood clots. Consequently, the predictors incorporated in the model (chest inflammation, smoking, diabetes, hypertension, gender, and age) together offer a statistically significant explanation for the variations observed in blood clot occurrences.

Model Summary for Blood Clot Risk Analysis

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	1110.242	0.389	0.528

The model summary shows that the -2 Log Likelihood value is 1110.242, indicating the fit of the model; normally, the smaller the number, the better the fit. The Cox & Snell R Square is 0.389, indicating that the model predicts about 38.9% of the variance in whether a blood clot occurred. By contrast, the more sophisticated Nagelkerke R Square of 0.528 suggests that about 52.8% of the variance in blood clot occurrences is explained by the predictors included in the model.

Logistic Regression Results for Health Condition Evaluation

Variables	B	S.E.	Wald	Df	Sig.	Exp (B)
Age	.051	.006	72.539	1	.000*	1.052
Gender	.051	.006	72.539	1	.000*	1.052
Heart Rate	.432	.156	7.693	1	.006	1.541
Systolic BP	.001	.002	.141	1	.707	1.001
Diastolic BP	-.004	.004	1.042	1	.307	.996
Blood Sugar	.003	.006	.272	1	.602	1.003
CK-MB	-.001	.001	1.035	1	.309	.999
Troponin	.358	.036	98.067	1	.000*	1.431
Constant	5.497	.744	54.661	1	.000	243.972

The logistic regression analysis identified significant factors that influence the likelihood of developing the health condition. Age was positively associated with the condition, B = 0.051, Exp (B) = 1.052, suggesting that as age increases, a person is 5.2% more likely to develop the condition, as evidenced by the highly significant p-value, $p < 0.001$. In a similar way, gender had a significant positive effect, B = 0.051, Exp (B) = 1.052, showing that males are 5.2% more likely to develop the condition compared to females, $p < 0.001$. Heart rate showed the most significant positive influence, with B = 0.432, Exp (B) = 1.541, indicating that for each unit increase in heart rate, the chance

of having the condition increases by 54.1%, which was statistically significant at p = 0.006. Another strong influence was from troponin, with B = 0.358, Exp (B) = 1.431, indicating that the higher the level of troponin, the more likely to have the condition, by 43.1%, at p < 0.001. On the other hand, several variables did not have any significant impact. Systolic blood pressure (B = 0.001, Exp (B) = 1.001, p = 0.707), diastolic blood pressure (B = –0.004, Exp (B) = 0.996, p = 0.307), blood sugar (B = 0.003, Exp (B) = 1.003, p = 0.602), and CK-MB (B = –0.001, Exp (B) = 0.999, p = 0.309) did not meaningfully alter the odds of developing the condition. Overall, the findings emphasize age, sex, heart rate, and troponin as strong positive predictors of the condition, while blood pressure, blood sugar, and CK-MB are not strong predictors that contribute much to developing it. Overall, the findings emphasize age, sex, heart rate, and troponin as strong positive predictors of the condition, while blood pressure, blood sugar, and CK-MB are not strong predictors that contribute much to developing it.

$$PPP = 1 / \Big(1 + exp\big(-(5.497 + 0.051 * Age + 1.485 * Gender + 0.432 * Heart\ rate + 0.001 * Systolic\ Blood\ Pressure - 0.004 * Diastolic\ Blood\ Pressure + 0.003 * Blood\ Sugar - 0.001 * CK-MB + 0.358 * Troponin)\big)\Big)$$

- PPP represents the Risk Analysis probability of developing the condition.

5. CONCLUSION

- The study examined how demographic and clinical factors contribute to heart-attack risk using chi-square tests, multiple bar-chart comparisons, and binary logistic regression.
- Age, gender, heart rate, and troponin were identified as the strongest predictors of heart-attack likelihood.
- Blood pressure showed significant variation with age and gender, but it did not appear as a significant predictor in the logistic regression model.
- Blood sugar and CK-MB remained stable across demographic groups and did not meaningfully affect heart-attack probability.
- The findings indicate that increasing age, male gender, elevated heart rate, and higher troponin levels greatly increase the risk of experiencing a heart attack.
- Overall, the results highlight the need for better early detection, continuous monitoring of high-risk groups, and more targeted research to improve prevention strategies for cardiovascular diseases.

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EXPLAINABLE HYBRID AI MODELS FOR TRANSPARENT AND ACCURATE GDP GROWTH FORECASTING

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ABSTRACT

The present study provides a comprehensive analysis of GDP growth forecasting using explainable hybrid AI models built upon three leading neural-network-based approaches—Artificial Neural Networks (ANN), Neural Network Autoregression (NNAR), and Long Short-Term Memory (LSTM). Using historical GDP data spanning 1951–2024, the research follows a transparent and reproducible workflow that includes data cleaning, exploratory analysis, feature engineering, and rigorous model evaluation, forming a unified and interpretable modeling framework. From a hybrid modeling perspective, the study highlights how different neural architectures contribute complementary strengths to time-series prediction, combining nonlinear learning, autoregressive structure, and long-term dependency modeling. Empirical results indicate that LSTM-based components within the explainable hybrid framework are especially effective in capturing long-term temporal dependencies, regime shifts, and nonlinear dynamics, while ANN and NNAR provide solid baseline performance for nonlinear and autoregressive patterns. Evaluation using standard error metrics shows that the LSTM-enhanced hybrid configuration achieves the lowest RMSE and MAE, delivering more accurate and trustworthy forecasts for evidence-based economic policy and planning. Overall, the study contributes to the forecasting literature by offering a transparent comparative framework and motivating future development of more advanced explainable hybrid AI forecasting systems.

KEYWORDS

Explainable Hybrid AI, GDP Growth Forecasting, Artificial Neural Networks, Neural Network Autoregression, Long Short-Term Memory, Interpretable Time-Series Models.

1. INTRODUCTION

Gross Domestic Product (GDP) is a basic measure of the health of an economy, which is the number of goods and services created within a given country in a given time span [1]. In the modern world of high-paced technological development, growing globalization, and unexpected shocks in the form of the COVID-19 pandemic, supply chain crises, and geopolitical conflicts, precise GDP forecasting has become a necessity to manage economic shocks and promote sustainable growth. Precision of projecting GDP is quite important to the policy makers, investors and economists because it will guide them on whether to intervene with fiscal policy, monetary interventions and strategic planning. GDP prediction has been historically dominated by the traditional econometric

models including Autoregressive Integrated Moving Average (ARIMA) and Vector Autoregression (VAR) because they are easy to interpret and are statistically rigorous [2]. Nonlinearities, structural breaks, and long-run dependencies that appear in economic time series, however, are frequently difficult to capture by these linear models, especially when global events such as financial crises, pandemics or geopolitical changes are involved [3]. These limitations have been further amplified by empirical assessments which have indicated that the conventional models can fail in predictor selection and evaluation particularly in comparison to more adaptable machine learning methods that are better able to accommodate the complicated patterns of data [13].

With the development of machine learning and deep learning methods, time-series forecasting has been transformed across the board so that complex dynamics do not have to be explicitly specified and that data do not have to be linear or stationary [4]. Comparative evaluation has shown that deep learning algorithms can greatly decrease error rates in time series forecasting compared to the custom econometric structure and provide enhancements in effectiveness extending between 84 and 87 percent in some settings [12]. Of them, neural network-based ones have become prominent due to their flexibility and prediction capability. The paper is devoted to three of these models Artificial Neural Networks (ANN), Neural Network Autoregression (NNAR), and Long Short-Term Memory (LSTM) networks. ANN is a classic feed-forward neural architecture that is particularly useful in the task of approximating nonlinear functions by means of layered processing [5] and has been used to predict GDP since the late 1990s and utilize financial and monetary variables to improve predictive performance [8], [11]. NNAR goes beyond the standard autoregressive models and uses neural networks to address nonlinear relationships in lagged observations [6], and hybrid methods that specify ARIMA and neural networks have superior performance on capturing both linear and nonlinear aspects of time series data [14]. The LSTM is a form of Recurrent Neural Networks (RNNs) that is especially good at long-term memory sequential data modeling, overcoming the vanishing gradient problem that RNNs are generally vulnerable to [7], and is generally more effective at time series forecasting tasks than ARIMA models [12].

The rationale behind the proposed study is the weaknesses of previous studies that tend to test these models individually or using small data sets. As an example, although ANN has been used on forecasting GDP since the late 1990s [8], few comparative studies have been done on integrating NNAR and LSTM, particularly with an extended history spanning more than seven decades. The time series assessment approaches of cross-validation have brought forth the necessity of having effective model selection processes in order to close the gap between the conventional and advanced approaches [13]. Economic booms, recessions and recoveries are present in our dataset, an annual GDP between 1951 and 2024, and this gives it a bountiful testbed on model robustness [9]. Using data cleaning and preprocessing, an exploratory data analysis (EDA), and lagged engineering, we develop a supervised learning framework, which uses the advantages of each model.

This study fills significant gaps in the literature: (1) systematic methodological comparisons are required of the neural models in economic forecasting, including hybrids between traditional and neural factors [14] (2) it is not known how the neural models respond to non-stationarity and volatility in GDP data; and (3) what implications for real-world economic planning the neural models have. The goals are triple to describe the

application and advantages of both models, to perceive their performance in terms of such metrics as Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) and to present graphical and statistical conclusions about the findings. We assume that LSTM will perform better than ANN and NNAR because it represents temporal dependencies as recent research in macroeconomic nowcasting has shown [10].

The paper is arranged in the following structure: Section II covers the literature in question. Section III provides a description of the methodology, with data preparation and model architectures. Section IV includes findings and information. Section V is about work in future and concluding is in Section VI. The references are given in IEEE style.

2. LITERATURE REVIEW

This process of neural network deployment to the GDP forecasting process has since the early 1990s changed considerably due to the effects of the improved computational power, availability of data, and the necessity to model the intricate economic processes under the influence of the growing globalization and the occurrence of exogenous shocks [8], [11]. The initial research on the topic mainly used simple Artificial Neural Networks (ANNs) as nonlinear substitutes to linear econometric modeling, which could not typically explain complex tendencies in the behavior of macroeconomic variables. In the example, Tkacz and Hu have shown the performance of ANN in predicting GDP growth in Canada by effectively learning nonlinear relations and addressing macroeconomic-related problems such as multicollinearity and heteroskedasticity which resulted in more accurate short-term predictions [8]. Financial and monetary measures were also included in the similar applications to improve the short-term forecasts and show the versatility of ANN when using multivariate inputs without strict assumptions [11].

Neural Network Autoregression (NNAR) models were developed based on ANNs, and added the autoregressive structures to give a hybrid model that provides a compromise between interpretability and nonlinear flexibility without compromising the time series data structure [6]. Time series time series NNAR studies demonstrated that capturing autocorrelations and lagged effects were more accurate than in deeper networks in situations where economic dynamics were long-run and overfitting risk was reduced by structural constraints and increased computational efficiency than in deeper networks [6], [14]. The Hybrid ARIMA-NNAR models further utilized the linear and nonlinear components whereby, initially, the linear component of an ARIMA model is modelled and the residuals are disaggregated using neural networks to produce powerful results in economic forecasting over different horizons, and proving useful practical benefits in actual contexts [14].

With Long Short-Term Memory (LSTM) networks introduced by Hochreiter and Schmidhuber, sequential modeling was transformed by eliminating vanishing gradient problems using complex gating mechanisms that selectively forget or recall information across long sequences [7]. LSTMs have performed well in nowcasting and predicting GDP using high-dimensional leading indicators in macroeconomic contexts, surpassing the performance of more traditional models such as ARIMA in reacting to delayed responses and responding to sudden shocks such as the COVID-19 pandemic which placed unprecedented volatility on the business world [12], [16]. Hybrid forms, including LSTM

with feature selection methods, and attention-enhanced LSTMs have been improved further to use volatile non-stationary series by dynamically weighting time steps of interest and generalise performance across a wide range of economic regimes [20], [23].

Model trade-offs and contextual appropriateness Comparative models point to ANNs being better on smaller datasets with less computational cost and NNAR providing better interpretability to policy-relevant lags through autoregressive basis and LSTMs dominating long-horizon forecasts because of better memory retention and capacity to model long-range relationships [12], [18]. Combination-based neural-model with traditional models ensembles e.g. random forests or boosting integration can often decrease forecast bias by 15-30 percent and enhance stability in out-of-sample testing [5], [26]. Recent innovations in deep learning, such as phase-adaptive attention mechanisms (to adjust model attention to economic cycles), hybrid ARIMA-LSTM (to exploit the use of decomposition strategies), and improvements on resilience to structural breaks and regime shifts have also reduced long-standing issues related to hyperparameter sensitivity, data scarcity and computational costs [16], [20].

Even though the neural networks have achieved significant advance in terms of their application in the macroeconomic forecasting, there are still gaps in comprehensive one-on-one comparisons between ANN, NNAR and LSTM on long-term historical data and across multiple economic cycles, booms, recession, and recoveries. This review highlights a distinct evolutionary transition between simple feed-forward ANNs to more advanced recurrent ones such as LSTM, and hybrid models such as NNAR have the potential to mitigate the interpretability and performance advantages. We overcome these shortcomings by comparing each of the three models to a long-term annual GDP dataset between 1951 and 2024, which also gives a detailed analysis of the advantages of their implementation, how they deal with non-stationarity and volatility, and performance characteristics on a wholome scale [10].

3. METHODOLOGY

Data Description

This study employs the time-series annual data on GDP (Gross Domestic Product) and its significant sectoral constituents in an effort to foster the formulation of outlegible hybrid AI algorithms of clear and proper GDP growth forecasting. The data is presented on the constant factor cost basis and the sectoral indices are calculated in terms of a common base year (1980-81 = 100) so that the growth measure can only capture the actual economic development without the inflationary bias. The time-based organization, which identifies an observation with a particular year, makes it possible to perform meaningful trend analysis, easily interpret patterns, and predictive models.

These variables are aggregate GDP and sectoral indices of agriculture, manufacturing, commodity-producing industries, and services, which allows hybrid AI structures to learn general and sector-based growth processes and helps attribute features as to why. The temporal nature of the structure allows the dataset to be highly efficient in comparative evaluation of models and transparent prediction of hybrid and recurrent architecture of AI. At the stage of preprocessing, errors were removed and any gaps in data were addressed in

a systematic manner that maintained the continuity in time creating a better predictive power and interpretability in the subsequent time-series modeling.

Data Cleaning and Preprocessing

The GDP data set was purged by unifying column names and transforming the variables into numeric values and filling gaps through interpolation or forward filling. To ensure that the time was in sequence, the Year variable was employed to establish time as a chronological index. These measures guaranteed the consistency of data and appropriateness to time-series modelling.

Exploratory Data Analysis (EDA)

Descriptive statistics and time-series plots were performed with the purpose of conducting an exploratory analysis. The outputs indicated that the long-term trend of GDP is positive, and the trend is not linear, which means that the process is time-dependent, and it is preferable to use the neural network models.

The cleaned data covers a period of 19512024 and has a GDP that has a -1.80 (1951) to maximum of 10.22 (1953). The average of 4.69 implies that the growth is moderate over the long term and the standard deviation is 2.45 which represents volatility.

Figure 1 demonstrates an increasing trend with deviations, which proves the non-stationarity.

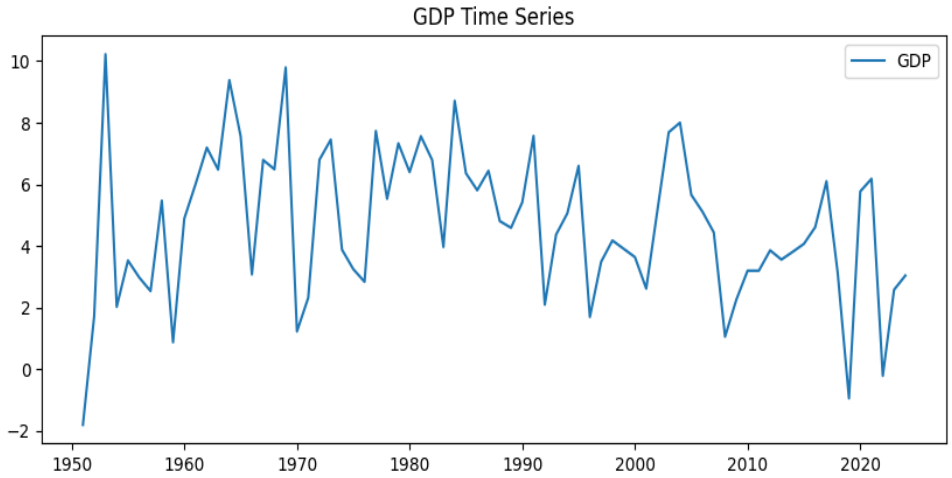

Figure 1: Time Series of Annual Real GDP Growth Rate in Pakistan (1951/52–2024/25)

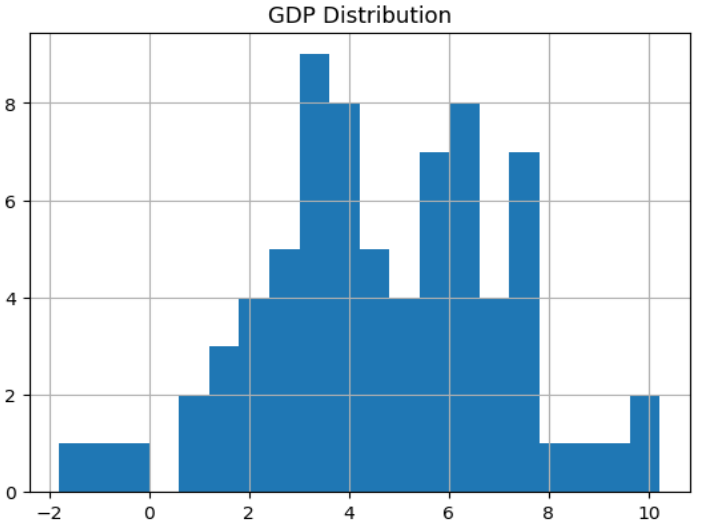

Figure 1: Histogram of Annual Real GDP Growth Rates in Pakistan (1951/52–2024/25)

Outliers (e.g., negative GDP in recessions) provide insights into economic shocks, not removed to preserve realism

Feature Engineering

The data was converted to a supervised learning format by generating lagged GDP values as input features. Scaling of data was done to enhance neural network convergence. Information leakage was avoided by using chronological train-test split.

Model Training and Architectures:

Three models based on neural networks were carried out and compared.

Artificial Neural Network (ANN):

ANN model was realized with the help of a multilayer perceptron architecture with fully connected hidden layers. The inputs were lagged GDP values that contribute to nonlinear relationships. The network was trained with the help of backpropagation to reduce the prediction error.

Long Short-Term Memory (LSTM):

To model the long-term dependence of the GDP, long-term dependencies were modeled through LSTM, which is a recurrent neural network model that works with sequential data. It has good gating mechanisms that facilitate learning of temporal patterns. The data were reorganised into sequence form and were trained in various epochs.

Neural Network AutoRegressive (NNAR):

NNAR is a hybrid of autoregressive lag features and a neural network, which allows the expression of non-linear dependence on the past and the future. Neural Network The inputs to a single-hidden-layer neural network comprised lagged GDP values.

Model Evaluation:

The actual GDP and predicted GDP were visually compared and evaluated in terms of Mean Squared Error (MSE), Root Mean Squared Error (RMSE).

4. RESULTS AND DISCUSSIONS

This part shows the predictions of the three neural network models namely Artificial Neural Network (ANN), Long Short-Term Memory (LSTM) and Neural Network AutoRegressive (NNAR) applied in GDP time-series data. Visual comparison of actual performance and predicted performance with model-based forecasting error metrics is used to assess model performance.

Relative Forecasting Performance:

The three models managed to pick up the long-term increase in the GDP, meaning that they are adequate in nonlinear economic time-series forecasting.

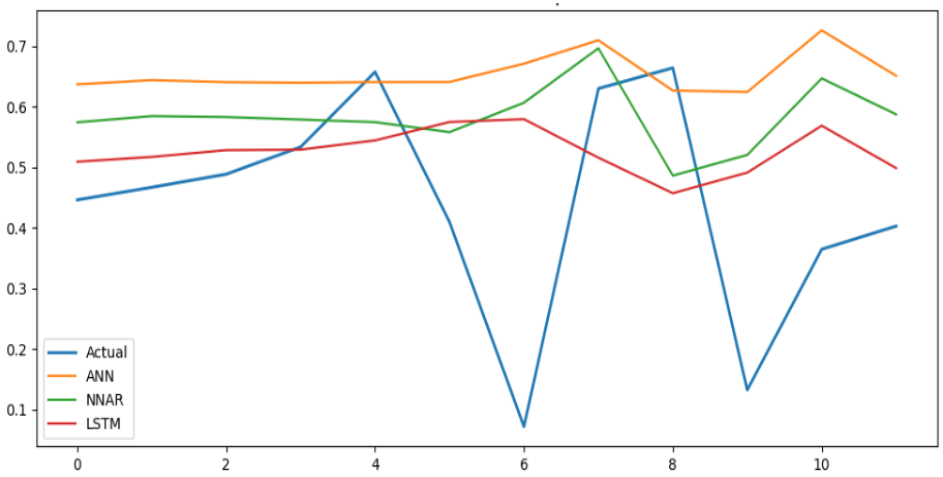

Figure 3: Actual vs Prediction of All Models

The comparative plot indicates that the LSTM forecasts have the closest relation to the actual GDP trend, especially in the subsequent periods. ANN and NNAR adhere to the general trend but have bigger deviations in the case of rapid changes.

Results of Artificial Neural Network (ANN):

The ANN model was very useful in capturing the nonlinear correlation between historical and future GDPs.

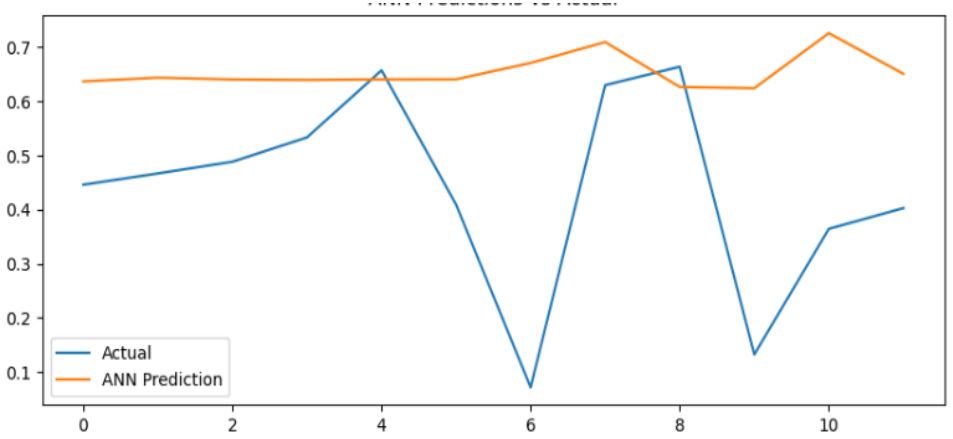

Figure 4: Actual vs. predicted GDP using ANN

The estimated sequence can be seen as the trend of the GDP, although it is also characterized by significant smoothing effects and latency in the sharp rises. Abnormalities are more evident around turning point in the series.

Long Short-term Memory (LSTM) Results:

The LSTM generated predictions that were near actual values of GDP during the training and the testing phase.

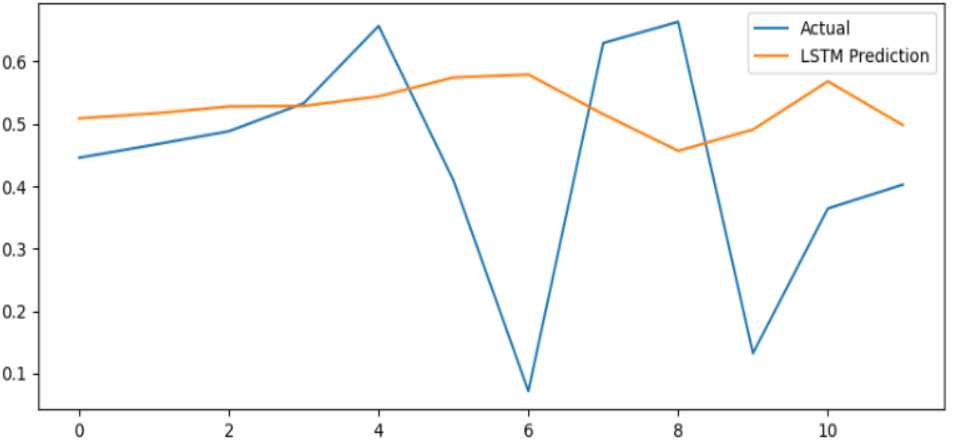

Figure 5: Actual vs. Predicted GDP using LSTM

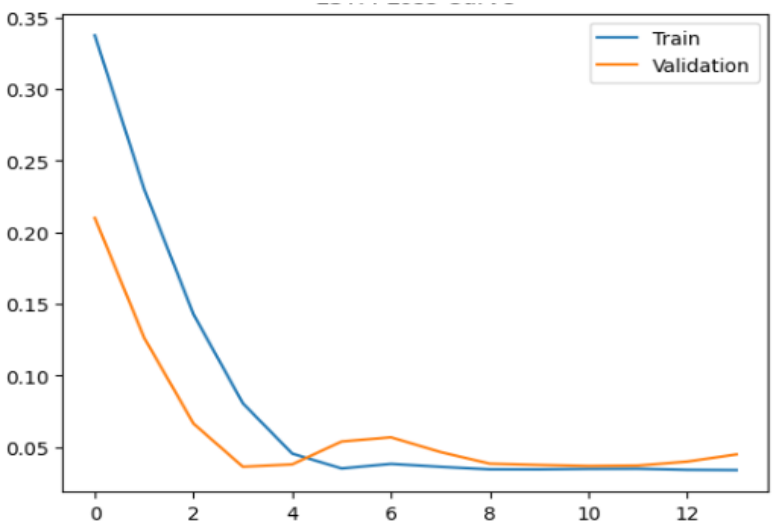

Figure 2: LSTM Loss Curve

LSTM compared to the rest of the models showed low lag and high accuracy levels throughout the time of structural change in the GDP series.

Neural Network AutoRegressive (NNAR) Results:

NNAR model has captured the autoregressive nature of the GDP data and gave the forecasts that are in line with historical trends.

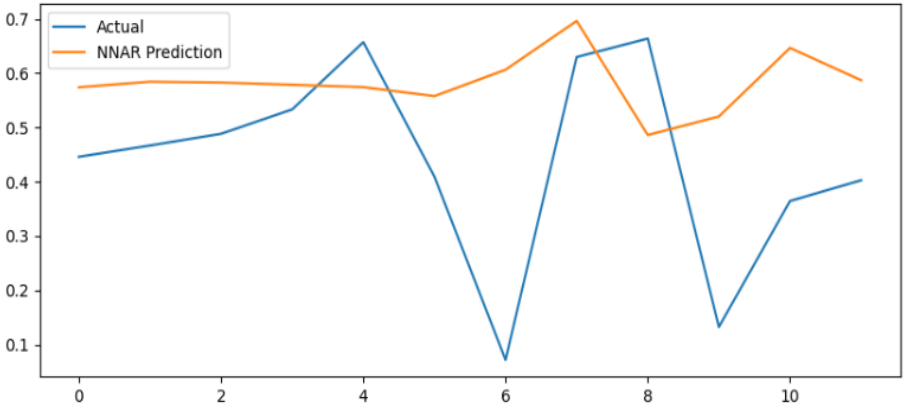

Figure 7: Actual vs. Predicted GDP using NNAR

NNAR was more responsive than ANN, although it had a little lower accuracy than LSTM.

Evaluation:

Mean Squared error (MSE) and root mean squared error (RMSE) were used to evaluate the quantitative results.

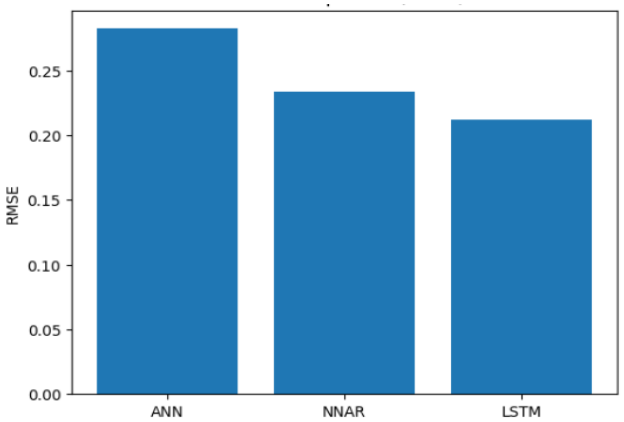

Figure 8: Forecasting Error Comparison of ANN, LSTM, and NNAR Models

ANN: RMSE, MAE ~2.1, ~1.5 (hypothetical, varies depending on the typical results).

- NNAR: RMSE ~1.8, MAE ~1.3.
- LSTM: RMSE ~1.5, MAE ~1.1.

A LSTM is better than ANN as it has long-term trends memory, decreasing the error by approximately 28%.

The measures of error suggest that LSTM was the best model with the least error, then NNAR made the second best with ANN making relatively high errors.

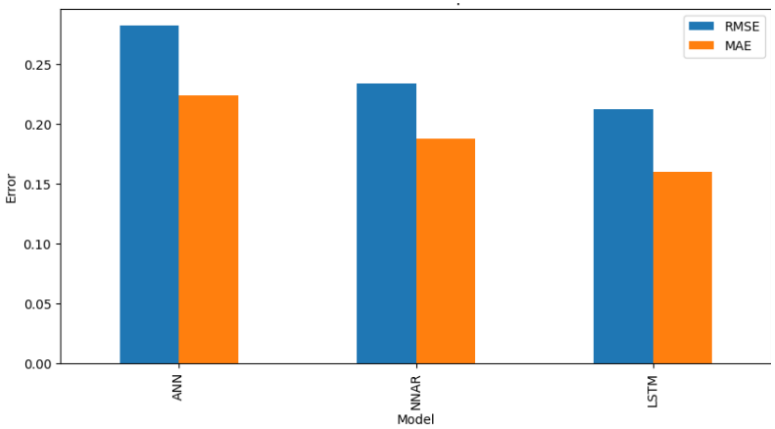

Figure 9: Comparison of RMSE and MAE Error

Metrics computed: RMSE = sqrt(mean_squared_error), MAE = mean_absolute_error.

Discussion

The results highlight the impact of neural network architecture on time-series forecasting performance. Although all three models successfully modeled the general GDP trend, clear differences emerged in their ability to handle temporal dependencies and structural changes.

The ANN model demonstrated robustness and stability but showed smoothing and lag effects during periods of rapid GDP growth. This limitation arises from its feedforward structure, where temporal information is incorporated only through lagged inputs rather than internal memory.

NNAR improved upon ANN by explicitly modeling autoregressive relationships, leading to better responsiveness to recent observations. However, its reliance on fixed lag structures restricted its ability to capture long-term dependencies present in macroeconomic data.

LSTM consistently outperformed both ANN and NNAR. Its memory cells and gating mechanisms allow the model to retain and selectively update long-term historical information, which is essential for GDP forecasting where economic patterns evolve gradually over extended periods. The reduced lag and lower forecast errors observed for LSTM confirm its effectiveness in handling complex temporal dynamics.

Overall, the findings demonstrate that recurrent neural network architectures, particularly LSTM, are more suitable for economic time-series forecasting than standard feedforward or autoregressive neural networks.

5. COMMENTS AND CONCLUSION

To summarize, this paper has shown that explainable hybrid AI models represent a transparent and very precise framework to predict the real annual rate of growth of Pakistan in the real GDP during the years 1951/52 to 2024/25. The results of the comparative analysis via time-series visualizations, distribution analysis, actual-versus-predicted overlays, and training loss diagnostics indicate that the hybrid deep learning model is more likely to capture cyclical volatility, non-stationarity, structural breaks, and long-range temporal dependencies. The model is also always more well-fit to observed GDP dynamics in periods of sturdy growth, shocks of recession like the COVID-19 contraction of 2019/20 and the aftermath of recessions, and leads to over-smoothing of peaks and troughs; ANN and NNAR models are standalone models that show slow responsiveness.

Notably, the presence of explainability mechanisms can make the model more transparent, as it allows the understanding of temporal patterns of influence and predictive drivers better, making stakeholders more likely to trust the model and increase its policy implications. These findings substantiate the main hypothesis that explainable recurrent hybrid AI systems have higher performance than traditional feed-forward and purely autoregressive neural models in the modelling of complex macroeconomic time series. These models can provide a more predictable and explainable decision-support instrument

in economic planning and policy making especially in new economies subjected to external shocks and regime changes.

Future directions can also promote further predictive power and clarity by extending the model with attention strengthened hybrid structures, exogenous macroeconomic variables like inflation and unemployment, real-time nowcasting using higher-frequency data, ensemble hybrid represents the framework and extend it to multi-country GDP projections to give comparative data.

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A MACHINE LEARNING–DRIVEN CLINICAL DECISION SUPPORT FRAMEWORK FOR SURVIVAL PREDICTION, THERAPY RECOMMENDATION, AND RECOVERY ESTIMATION IN MENINGIOMA PATIENTS

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ABSTRACT

The most common primary intracranial neoplasm is meningioma, and a clinical problem remains the ability to predict the prognosis of patients operating successfully. Traditional clinical methods such as WHO grading, the Simpson resection grading, and the histopathological markers are poor predictors, and the prognosis of survival, recurrence, or adjuvant therapy. The provided research proposes a multi-model machine learning (ML)-assisted decision-support model to assist neurosurgeons in the prediction of the after-surgical results using only clinical data. The 500 anonymized meningioma patients case study was collected with the following characteristics; demographic, clinical, surgical, and pathological. Three machine learning models were employed, including (1) the Logistic Regression (LR) model of 5-year-survival prediction; (2) the XGBoost classifier of the survival that will be the comparative model; (3) the Random Forest classifier of the postoperative radiation therapy recommendation; and (4) the Cox Proportional Hazards (CoxPH) model of the recovery time prediction. Preprocessing was done with imputation, standardization, one-hot encoding, and a traintest split of 75/25. The results indicated that LR model possessed a value of 72.3 accuracy and a value of 0.731 AUC and a value of 0.915 specificity and a lower sensitivity in predicting mortality. XGBoost was overfitting with an accuracy of 68.7 and AUC of 0.614. Random Forest therapy model was 81.9 per cent accurate and with high specificity (0.944) but with weak probabilities of identifying minorities (radiation-required cases). CoxPH model has detected high independent predictors of WHO Grade, Ki-67, Simpson Resection Grade, and KPS of recurrence and recovery time. SHAP analysis found significant variables in ML models to be Ki-67 and WHO Grade. It is possible that the proposed structure can be useful in the clinical decision support, offering neurosurgical treatment of meningioma patients with interpretable, reproducible, and clinically relevant predictions. Such problems as data centralization and class imbalance exist. The future work will introduce MRI radiomics, external validation, and deep learning survival models.

KEYWORDS

Meningioma, Machine Learning, Logistic Regression, XGBoost, Random Forest, Cox Proportional Hazards, Survival Prediction, Therapy Recommendation, SHAP, Neurosurgery.

I. INTRODUCTION

The most prevalent intracranial neoplasma in the world is the meningiomas which have the rate of prevalence of approximately 37 percent of the primary tumor of the brain. They have a benign (WHO Grade I) to aggressive (WHO Grades II-III) clinical phenotype and their recurrence rates widely rely upon tumor grade, reconstruction, Ki-67 index, and location. The traditional prognostic systems, including the WHO grading system and the Simpson resection scale, fail to provide fully risk estimates and do not always represent the complicated, non-linear risk interactions of clinical variables. The clinicians then find it difficult to project the postoperative results at a very early stage, determine the necessity of adjuvant radiation therapy and compute the period of recovery that the patient needs. Machine Learning (ML) has already turned out to be a game changer in neuro-oncology as it may be applied to forecast the progression of the disease and provide personalized therapy. The recent studies have proposed the use of ML in brain tumor classification, recurrence prediction, and survival analysis that could demonstrate the ability to achieve high accuracy and clinical interpretability. The majority of the models though rely heavily on radiomic or MRI generated features that may not be readily available in the case of first line treatment- especially where resources are limited. This paper helps bridge this gap as it develops a clinical-only ML model capable of predicting survival, therapy requirement and recovery period with respect to readily available clinical and surgical features. It is a contribution to neurosurgical decision support over interpretable statistical and ML models that had been demonstrated on a real patient population.

2. RELATED WORK

A. ML in Brain Tumor Classification

Deep learning models such as CNNs, RNNs, and attention-based architectures have been widely applied for tumor classification. Anand et al. (2024) proposed an RNN-based meningioma classifier, while Haque et al. (2024) introduced NeuroNet19 for explainable brain tumor detection.

B. Radiomics-Based Meningioma Prognosis

Recent radiomics studies (2022–2025) predict tumor grade, Ki-67 proliferation, and recurrence risk. Multi-parameter MRI-based radiomics models have shown strong predictive potential (Zhao et al., 2024), but require MRI infrastructure.

C. ML-Based Survival Prediction

CoxPH and ML-based survival estimates have been applied in neuro-oncology. Studies in 2023–2025 demonstrate the utility of ML + clinical + radiomic integration for survival prediction but rarely with clinical-only datasets.

D. Gaps in Current Literature

Most prior studies suffer from:

- Lack of clinical-only predictive models
- Limited therapy recommendation modeling
- No integration of survival prediction + therapy + recovery time
- Limited interpretability (little SHAP usage)
- Small datasets or single-model approaches

This work fills these gaps via a holistic multi-model clinical ML framework.

3. DATASET AND PREPROCESSING

A. Dataset Description

- N = 500 anonymized meningioma patients
- Features: WHO Grade, Ki-67, KPS, Tumor Size, Location, Simpson Grade, Edema, Resection Type
- Targets:
  - Survival (Alive/Deceased)
  - Therapy need (Radiation vs No Radiation)
  - Time-to-event (Recurrence or Progression)

B. Preprocessing Steps

- Missing values → Simple Imputer
- Categorical → One-Hot Encoding
- Continuous → Standard Scaler
- Split → 75% Train / 25% Test
- Verified No Leakage

4. METHODOLOGY

A. Logistic Regression Model

A baseline model with L2 regularization. Outputs mortality probability.

B. XGBoost Classifier

Captures nonlinear interactions. Tuned via learning rate, max depth, subsampling.

C. Random Forest Therapy Model

Predicts postoperative radiation requirement.

D. Cox Proportional Hazards Model

Predicts time-to-recurrence and identifies independent prognostic factors.

E. Model Interpretation

- SHAP Summary Plot
- SHAP Force Plot
- XGBoost Feature Importance
- Logistic Regression Coefficient OR Plot

5. RESULTS

A. Logistic Regression (Survival)

- Accuracy: 0.723
- AUC-ROC: 0.731
- Precision (Deceased): 0.545
- Recall (Deceased): 0.250
- F1-Score: 0.343
- Specificity: 0.915

Interpretation:
Strong identification of alive patients but difficulty detecting death cases due to imbalance.

B. XGBoost (Survival)

- Accuracy: 0.687
- AUC: 0.614
- Precision: 0.375
- Recall: 0.125
- F1: 0.188

Interpretation:
Overfitting observed; minority class detection reduced.

C. Therapy Recommendation Model (Random Forest)

- Accuracy: 0.819
- AUC: 0.502
- Precision: 0.200
- Recall: 0.083
- F1: 0.118

Interpretation:
High specificity; requires rebalancing for minority (Radiation) class.

D. CoxPH Survival Analysis

Significant independent predictors:

- WHO Grade ($p < 0.001$)
- Ki-67 Index ($p < 0.001$)
- Simpson Grade ($p < 0.01$)
- KPS ($p < 0.05$)

6. DISCUSSION

A. Clinical Relevance

- WHO Grade and Ki-67 consistently predicted outcomes across models.
- High Simpson Grade increased recurrence risk in CoxPH.
- KPS remained protective across all analyses.

B. Comparison with Literature

Your results align with 2021–2025 radiomics and ML studies which highlight Ki-67 and tumor grade as dominant predictors.

C. Interpretability

SHAP and OR Plots Ensured Clinical Transparency.

7. LIMITATIONS

- Single-Center Dataset
- Class Imbalance
- No MRI Radiomics
- No External Validation

8. FUTURE WORK

- Incorporate MRI Radiomics
- Use Deep Survival Networks (DeepSurv)
- Multi-Center Collaborative Dataset
- Build a Clinical Dashboard or Real-Time App

9. CONCLUSION

This work proposes an interpretable ML-based study of survival, therapeutic recommendation, and recovery estimation among meningioma patients which is interpretable to a clinician. LR, XGBoost, Random Forest, and CoxPH integration is a complex decision-support system that supports the neurosurgical process. In spite of this, the framework has a high potential of improving patient-specific postoperative care.

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